UCONN
UNIVERSITY OF CONNECTICUT

Forces, Moments, and Free Body Diagrams (Part I)

ENGR 1166 Biomedical Engineering

Recap

A body at rest will remain at rest; a body in motion will move in a straight line with constant velocity unless a net external force acts upon it

$$\sum_i \vec{\boldsymbol{F}}_i = \boldsymbol{0}$$

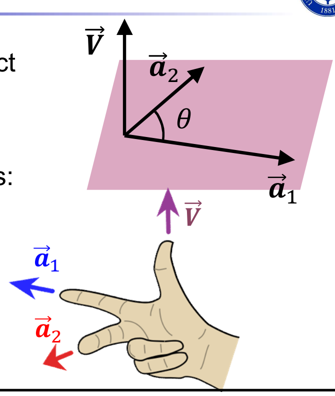
Recap

The cross product

$$\vec{\boldsymbol{V}} = \vec{\boldsymbol{a}}_1 \times \vec{\boldsymbol{a}}_2$$

The magnitude is:

$$V = a_1 a_2 \sin\theta$$



Equilibrium of a particle

Let us a consider a body of negligible dimensions (particle)

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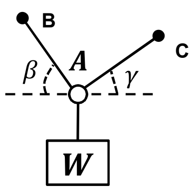

Particle A is pulled by the weight W
and supported by two wires (B, C)

Equilibrium of a particle

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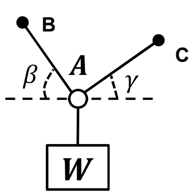
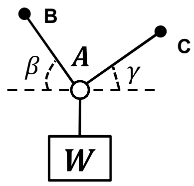


If the resultant (sum) of **all** the forces acting on the particle A is zero then the particle is **in equilibrium**

Particle A is pulled by the weight W
and supported by two wires (B, C)

Equilibrium of a particle

When is the particle A in equilibrium?



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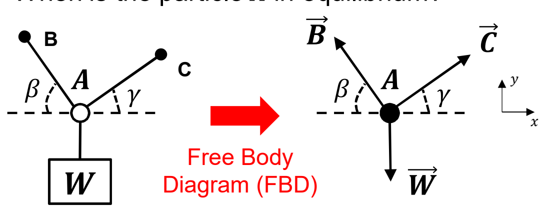
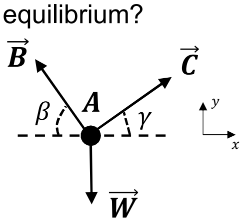


Equilibrium of a particle

When is the particle A in equilibrium?

$$\sum_i \vec{F}_i = \mathbf{0}$$

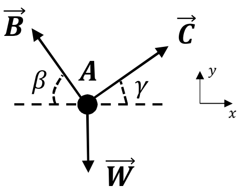


Equilibrium of a particle

When is the particle A in equilibrium?

$$\sum_i \vec{F}_i = 0$$

$$\vec{B} + \vec{C} + \vec{W} = 0$$



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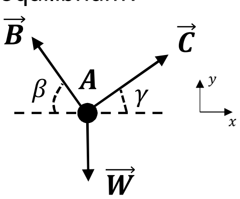
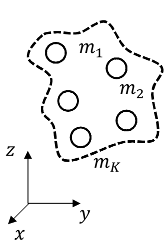


x-axis: $C\cos\gamma - B\cos\beta = 0$

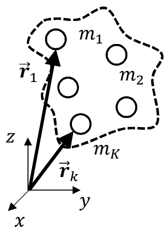
y-axis: $C\sin\gamma + B\sin\beta - W = 0$

Center of mass



Let us consider K particles in a volume:

Center of mass

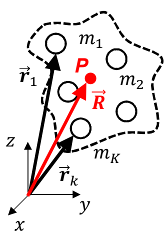


Let us consider K particles in a volume:

$m_i \stackrel{\text{def}}{=}$ mass of particle i, $i = 1,2,\dots,K$
$\vec{\boldsymbol{r}}_i \stackrel{\text{def}}{=}$ position of particle i, $i = 1,2,\dots,K$

$$M \stackrel{\text{def}}{=} \sum_{i=1}^{K} m_i$$

Center of mass



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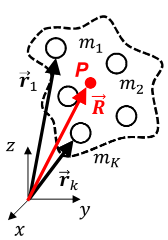
The center of mass (CoM) is the point P s.t.:

$$\vec{\boldsymbol{R}} = \frac{1}{M}\sum_{i=1}^{K} m_i \vec{\boldsymbol{r}}_i$$

Center of mass



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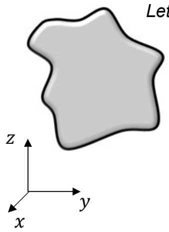
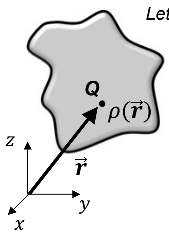
CoM is the mean location of a distribution of mass in space

Center of mass in a rigid body



Let us consider a rigid body occupying a volume V:

Center of mass in a rigid body



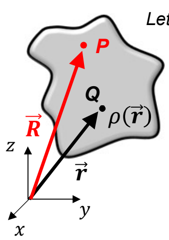
Let us consider a rigid body occupying a volume V:

$\vec{\boldsymbol{r}} \stackrel{\text{def}}{\equiv}$ position of generic point Q in V

$\rho(\vec{\boldsymbol{r}}) \stackrel{\text{def}}{\equiv}$ mass density at point Q

$M \stackrel{\text{def}}{\equiv} \int_V \rho(\vec{\boldsymbol{r}}) dV$

Center of mass in a rigid body



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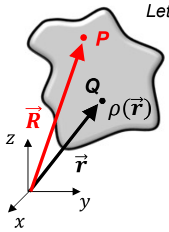

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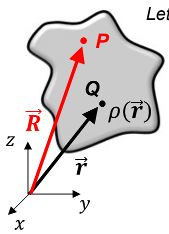
If $\rho(\vec{r}) = \rho \quad \Rightarrow \quad \vec{R} = \frac{1}{V}\int_V \vec{r}dV$ P is the centroid of the body

Center of mass in a rigid body



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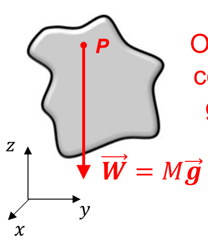
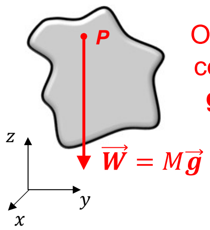
CoM is **NOT** the point at which a plane separates the body into two equal halves

Center of mass in a rigid body



On earth, the center of mass coincides with the **center of gravity** (CoG) of the body

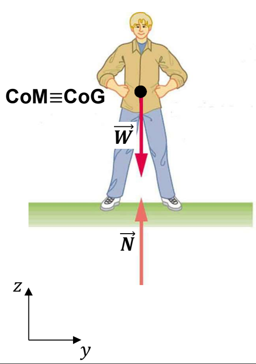
Center of mass in a rigid body



On earth, the center of mass coincides with the **center of gravity** (CoG) of the body

i.e., the point at which we consider all of the body's weight to be concentrated

CoM and CoG: equilibrum



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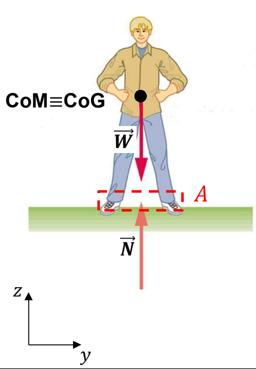

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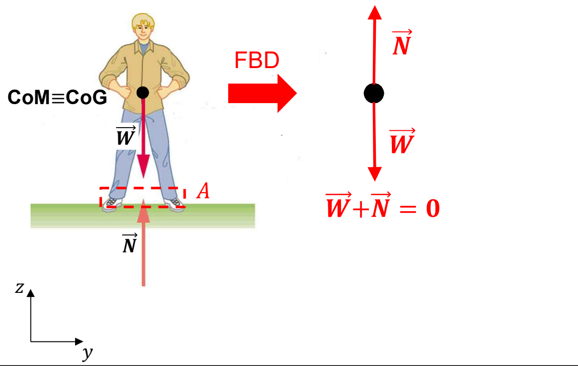
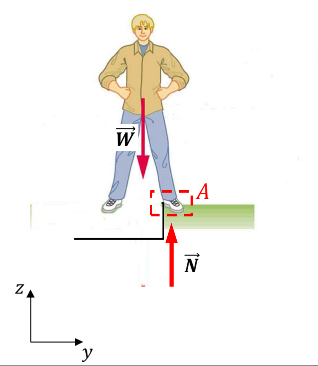


CoM and CoG: moment



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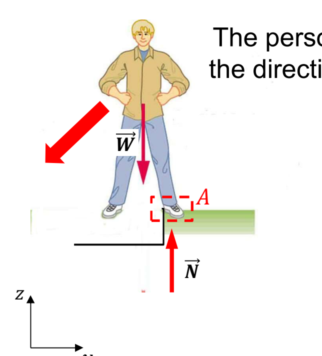
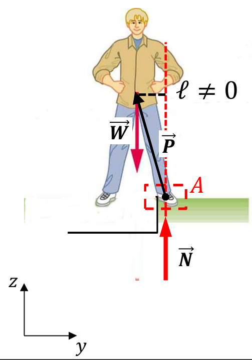


The person falls according to the direction of the bold arrow

CoM and CoG: moment



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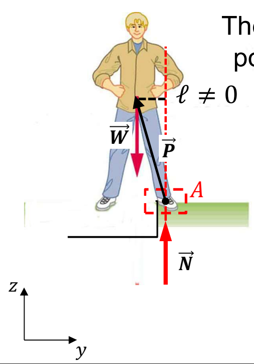
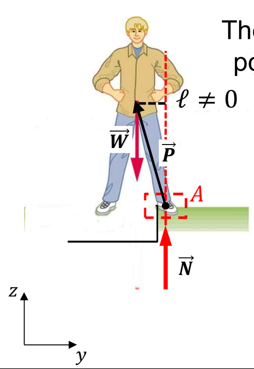


The **moment** of $\vec{W}$ about the point A is the cross product:

$$\vec{\tau}_{W,A} = \vec{P} \times \vec{W}$$

CoM and CoG: moment



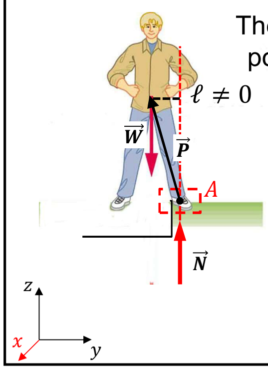
The **moment** of $\vec{W}$ about the point A is the cross product:

$$\vec{\tau}_{W,A} = \vec{P} \times \vec{W}$$

In practice, the magnitude of $\vec{\tau}_{W,A}$ is the magnitude of $\vec{W}$ times the perpendicular distance ℓ from the line of action to the point A

$$\tau_{W,A} = W\ell$$

CoM and CoG: moment



The **moment** of $\vec{W}$ about the point A is the cross product:

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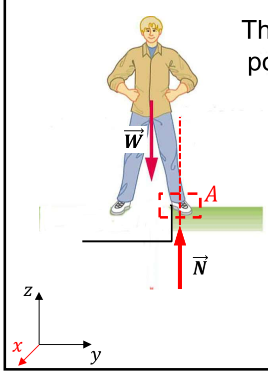
Remember, the direction of $\vec{\tau}_{W,A}$ is perpendicular to the plane of $\vec{W}$ and $\vec{P}$!

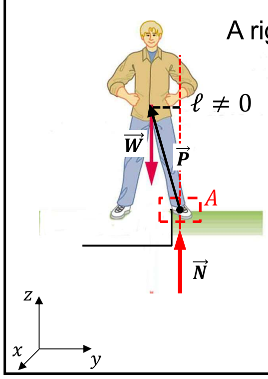
What about the moment of $\vec{N}$?



The **moment** of $\vec{N}$ about the point A is the cross product:

$$\vec{\tau}_{N,A} = \mathbf{0} \times \vec{N} = \mathbf{0}$$

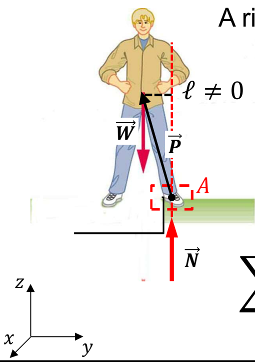
CoM and CoG: equilibrium



A rigid body is **in equilibrium** if two conditions hold:

1) The resultant of **all the forces** acting on the body is zero
2) The resultant of **all the moments** on the body is zero

CoM and CoG: equilibrium



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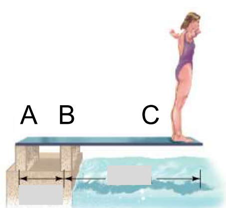
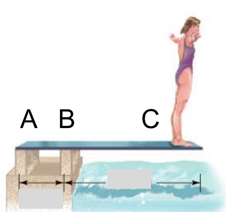
$$\sum_i \vec{\boldsymbol{F}}_i = \boldsymbol{0}; \quad \sum_i \vec{\boldsymbol{\tau}}_{i,A} = \boldsymbol{0}$$

Free body diagrams (FBDs)



*Let us assume that the entire system (person + diving board) is **in equilibrium***

Free body diagrams (FBDs)



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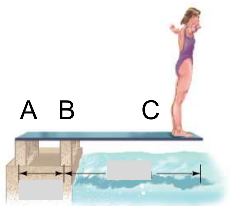
What component do we want to study?

Free body diagrams (FBDs)



*Let us assume that the entire system (person + diving board) is **in equilibrium***

What component do we want to study?

- ❑ Entire system
- ❑ Person only
- ❑ Diving board only

Case I: person only

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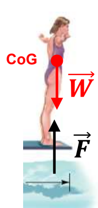
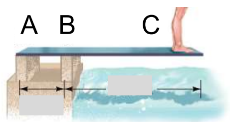


Forces applied on the person:

- ❑ Weight $\vec{W}$
- ❑ Reaction force $\vec{F}$ from the board

$$\vec{W} + \vec{F} = \mathbf{0}$$

Case II: diving board only



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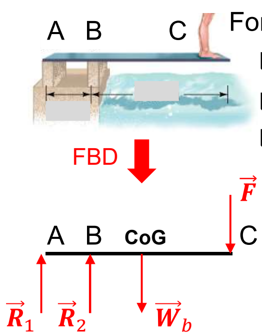
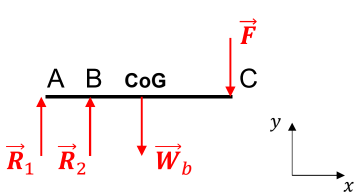


Forces applied on the board:

- ❑ Weight $\vec{\boldsymbol{W}}_b$
- ❑ Force $\vec{\boldsymbol{F}}$ from person
- ❑ Reaction forces at points A and B

Case II: diving board only

- ❑ x-axis: $F_x + W_{b,x} + R_{1,x} + R_{2,x} = 0$
- ❑ y-axis: $-F - W_b + R_1 + R_2 = 0$



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- ~~x-axis: $F_x + W_{b,x} + R_{1,x} + R_{2,x} = 0$~~
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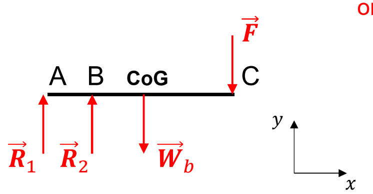
All the components on the x-axis are 0!



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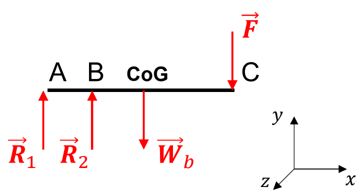
- x-axis: $F_x + W_{b,x} + R_{1,x} + R_{2,x} = 0$
- y-axis: $-F - W_b + R_1 + R_2 = 0$
- z-axis: $\sum \tau_i = 0$



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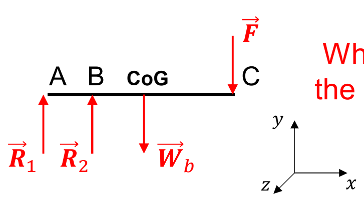
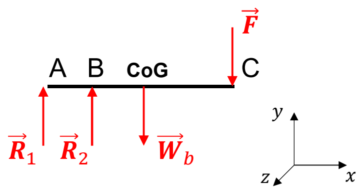


What point to take the moment about?

Case II: diving board only

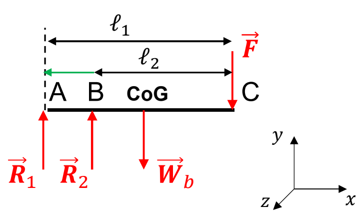
Let us take the moments about point B:



Case II: diving board only

Let us take the moments about point B:

- ❑ Moment of $\vec{R}_1$: $\tau_{R_1,B} = -R_1(\ell_1 - \ell_2)$



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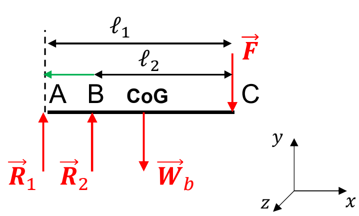
- ❑ Moment of $\vec{R}_1$: $\tau_{R_1,B} = -R_1(\ell_1 - \ell_2)$
- ❑ Moment of $\vec{R}_2$: $\tau_{R_2,B} = 0$



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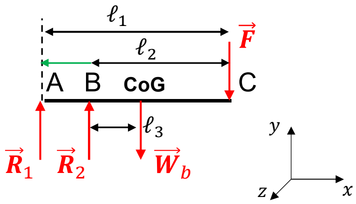
- ❑ Moment of $\vec{W}_b$: $\tau_{W_b,B} = -W_b\ell_3$



Case II: diving board only

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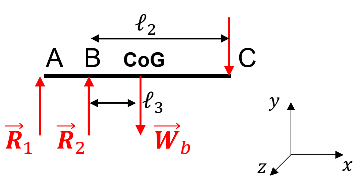
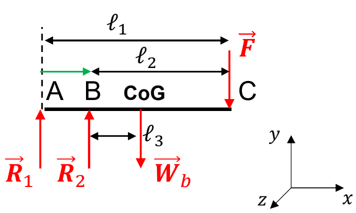
- ❑ Moment of $\vec{R}_1$: $\tau_{R_1,B} = -R_1(\ell_1 - \ell_2)$
- ❑ Moment of $\vec{R}_2$: $\tau_{R_2,B} = 0$
- ❑ Moment of $\vec{W}_b$: $\tau_{W_b,B} = -W_b\ell_3$
- ❑ Moment of $\vec{F}$: $\tau_{F,B} = -F\ell_2$



Case II: diving board only

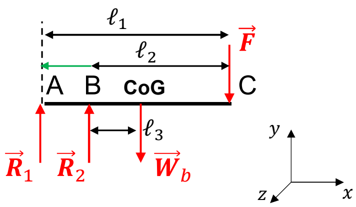
Let us take the moments about point A:

- ❑ Moment of $\vec{R}_1$:
- ❑ Moment of $\vec{R}_2$:
- ❑ Moment of $\vec{W}_b$: $\tau_{W_b,A} = -W_b(\ell_1 - \ell_2 + \ell_3)$
- ❑ Moment of $\vec{F}$:



Equilibrium of the diving board

- forces (y-axis): $-F - W_b + R_1 + R_2 = 0$
- moments: $\tau_{R_1,B} + \tau_{R_2,B} + \tau_{W_b,B} + \tau_{F,B} = 0$



Equilibrium of the diving board

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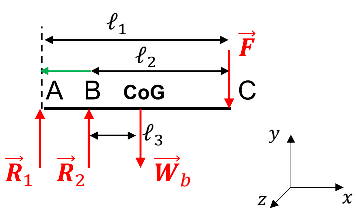

An example

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In summary…

General rules for FBDs include:

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- Show all forces and moments acting on $\mathcal{H}$ because of the bodies that $\mathcal{H}$ is detached from
- Do not forget to include $\mathcal{H}$'s own weight!
- For each force applied on $\mathcal{H}$ show (i) point of application, (ii) magnitude, (iii) direction

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An example (obvious from FBD)

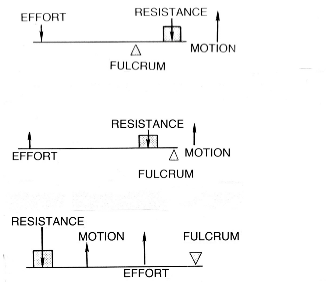
The body on the diving board is composed of numerous segments, each one having its own weight and CoG, but – in the FBD – we clearly see that we represented the body as producing a single force with a single CoG

Levers in free body diagram

class I

class II

class III



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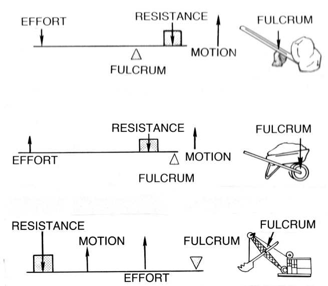

An example

Arm holding a weight

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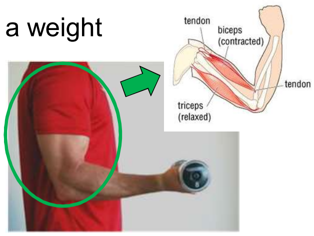
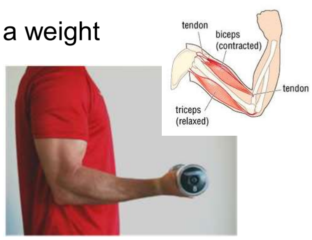


An example

Arm holding a weight



Let us assume that the lower arm forms a 90° angle with the upper arm and that the weight is applied to the lower arm

An example

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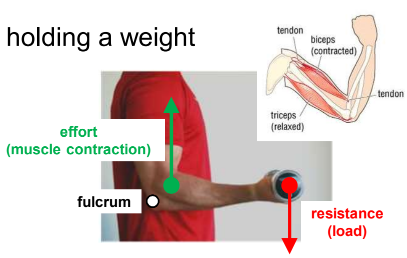

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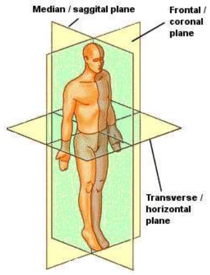
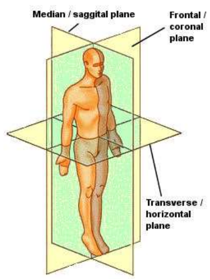
Positions defined in Anatomy



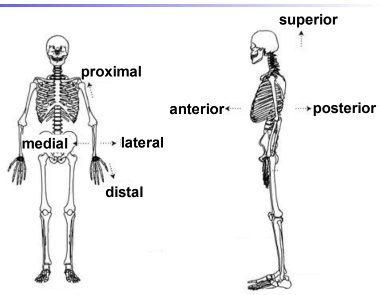
Positions defined in Anatomy



Standard body position

- ❑ Standing erect and facing forward
- ❑ Upper limbs hanging to the sides with palms facing forward

Positions defined in Anatomy



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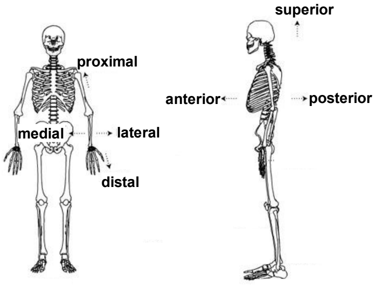

- ❑ **Medial:** towards the median plane
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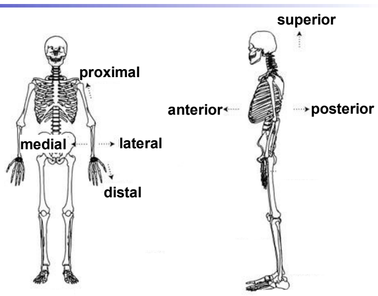

- ❑ **Medial:** towards the median plane
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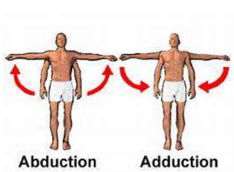
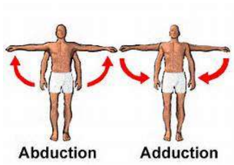
- ❑ **Proximal:** closer to the trunk
- ❑ **Distal:** away from the trunk
- ❑ **Anterior:** towards the front
- ❑ **Posterior:** towards the back
- ❑ **Superior:** towards the head
- ❑ **Inferior:** towards the feet

Movements in Anatomy

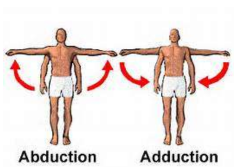


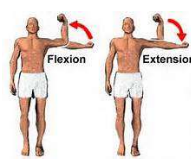
Movements in Anatomy



- **Abduction:** moves a body part *away from* the midline
- **Adduction:** moves a body part *toward* the midline

Movements in Anatomy





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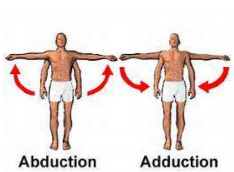
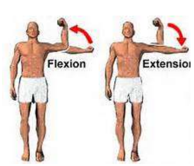




- **Abduction:** moves a body part *away from* the midline
- **Adduction:** moves a body part *toward* the midline
- **Flexion:** *decreases* the angle between body parts
- **Extension:** *increases* the angle between body parts

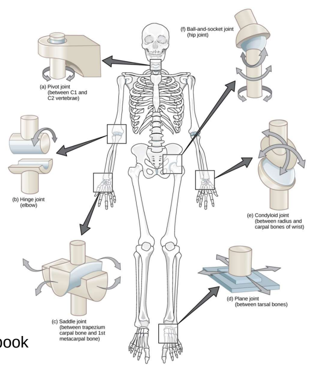
Anthropomorphic data

TABLE 4.1 Anthropomorphic Data

Segment	Definition	Segment Weight/ Body Weight	Center Mass/ Segment Length	
			Proximal	Distal
Hand	Wrist axis/knuckle II middle finger	0.006	0.506	0.494
Forearm	Elbow axis/ulnar styloid	0.016	0.430	0.570
Upper arm	Glenohumeral axis/elbow axis	0.028	0.436	0.564
Forearm and hand	Elbow axis/ulnar styloid	0.022	0.682	0.318
Total arm	Glenohumeral joint/ulnar styloid	0.050	0.530	0.470
Foot	Lateral malleolus/head metatarsal II	0.0145	0.50	0.50
Leg	Femoral condyles/medial malleolus	0.0465	0.433	0.567

Source: your textbook

Joint movements



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