UCONN
UNIVERSITY OF CONNECTICUT

Forces, Moments, and Free Body Diagrams (Part II)

ENGR 1166 Biomedical Engineering

Recap

A rigid body is **in equilibrium** if (1) the resultant of all the forces acting on the body is zero and (2) the resultant of all the moments about a given point A on the body is zero

$$\sum_i \vec{\boldsymbol{F}}_i = \boldsymbol{0}; \quad \sum_i \vec{\boldsymbol{\tau}}_{i,A} = \boldsymbol{0}$$

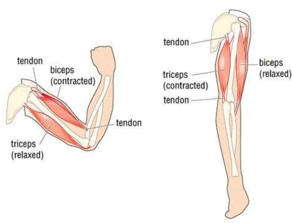
Recap

- A free body diagram (FBD) is a pictorial device to analyze the forces and moments acting on a body
- The body may consist of many components, each one acting as a single body. If so, a whole series of FBDs may be necessary
- In a FBD, constraints are replaced by arrows representing the forces and moments they generate

Case study: the human arm

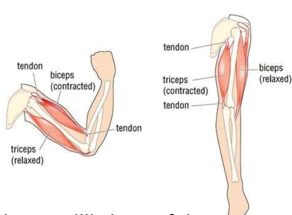
Case study: the human arm

muscles and tendons:



Case study: the human arm

muscles and tendons:



We want to study the equilibrium of the system composed by the forearm and the hand when a load is held

Step 1: what assumptions?

Step 1: what assumptions?

- The analysis is in 2D (single plane)

Step 1: what assumptions?

- The analysis is in 2D (single plane)
- The forearm and hand are a single rigid body

Step 1: what assumptions?

- ❑ The analysis is in 2D (single plane)
- ❑ The forearm and hand are a single rigid body
- ❑ The angle formed between the upper arm and the forearm is 90°

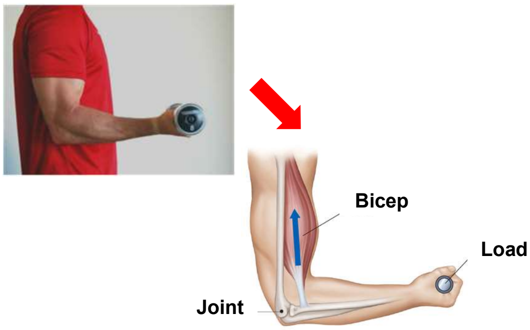
Step 1: what assumptions?

- ❑ The analysis is in 2D (single plane)
- ❑ The forearm and hand are a single rigid body
- ❑ The angle formed between the upper arm and the forearm is 90°
- ❑ The load is applied at the midpoint of the hand

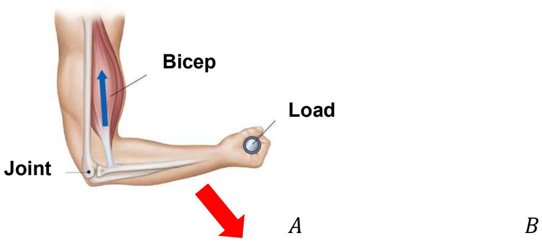
Step 1: what assumptions?

- ❑ The analysis is in 2D (single plane)
- ❑ The forearm and hand are a single rigid body
- ❑ The angle formed between the upper arm and the forearm is 90°
- ❑ The load is applied at the midpoint of the hand
- ❑ Only the Bicep muscle is considered

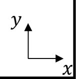
Step 1: outcome



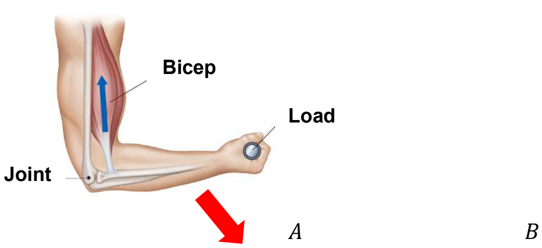
Step 2: free body diagram



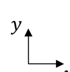
$AB \stackrel{\text{def}}{=}$ forearm+hand



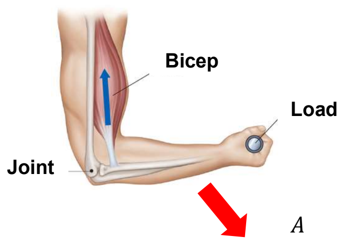
Step 2-a: what forces?

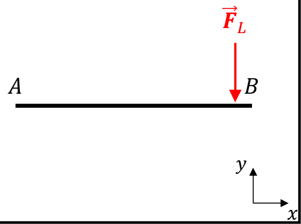


$AB \stackrel{\text{def}}{=}$ forearm+hand



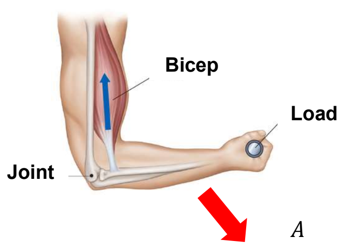
Step 2-a: what forces?

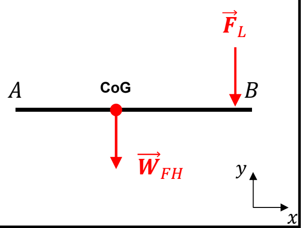




$AB \overset{\text{def}}{=}$ forearm+hand
$\vec{F}_L \overset{\text{def}}{=}$ weight of the load

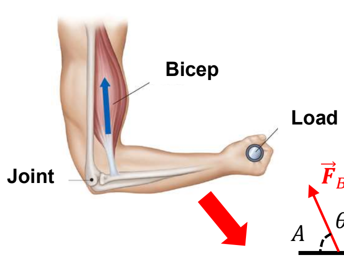
Step 2-a: what forces?

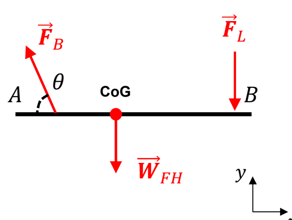




$AB \overset{\text{def}}{=}$ forearm+hand
$\vec{F}_L \overset{\text{def}}{=}$ weight of the load
$\vec{W}_{FH} \overset{\text{def}}{=}$ weight of the forearm+hand

Step 2-a: what forces?

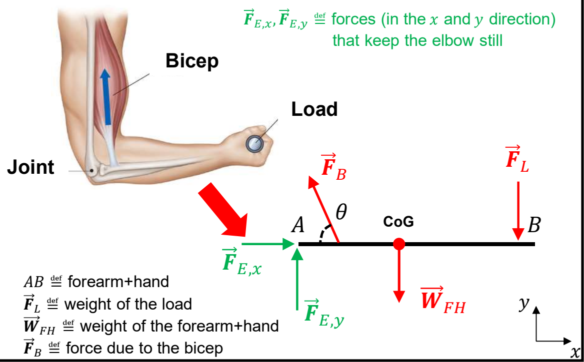




$AB \overset{\text{def}}{=}$ forearm+hand
$\vec{F}_L \overset{\text{def}}{=}$ weight of the load
$\vec{W}_{FH} \overset{\text{def}}{=}$ weight of the forearm+hand
$\vec{F}_B \overset{\text{def}}{=}$ force due to the bicep

Step 2-a: what forces?

$\vec{F}_{E,x}, \vec{F}_{E,y} \stackrel{\text{def}}{=}$ forces (in the x and y direction) that keep the elbow still

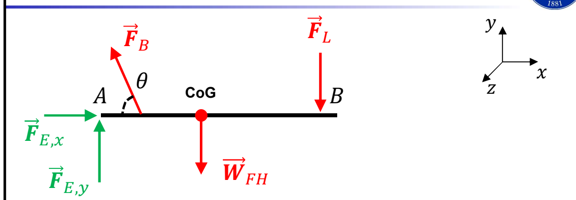


$AB \stackrel{\text{def}}{=}$ forearm+hand
$\vec{F}_L \stackrel{\text{def}}{=}$ weight of the load
$\vec{W}_{FH} \stackrel{\text{def}}{=}$ weight of the forearm+hand
$\vec{F}_B \stackrel{\text{def}}{=}$ force due to the bicep

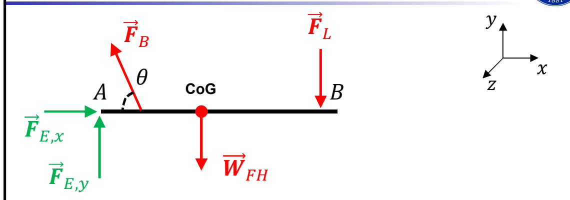
A note on forces $\vec{F}_{E,x}, \vec{F}_{E,y}$

- Include forces experienced between segments at the articulating surfaces
- Include the effect of muscle contraction (e.g., compressive, possibly shear and torsional forces)
- Cannot be directly measured. Hence, we measure them indirectly by using both kinematic and anthropometric data

Step 2-b: where are forces applied?

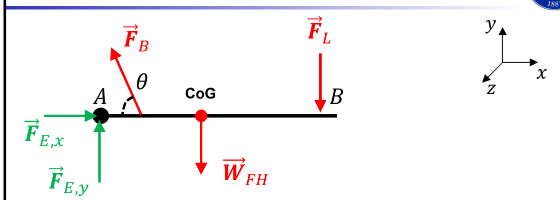


Step 2-b: where are forces applied?

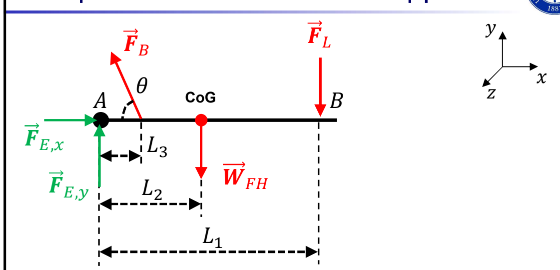


Because the study of equilibrium will require the analysis of the moments, let us first choose a reference point (e.g.,A) and measure the distance of the forces' application points from it

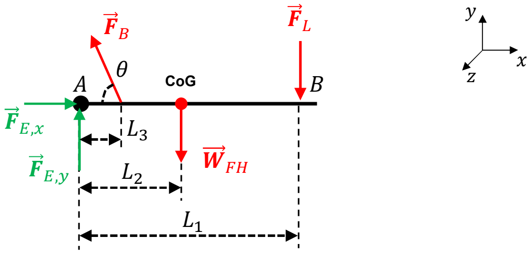
Step 2-b: where are forces applied?



Step 2-b: where are forces applied?

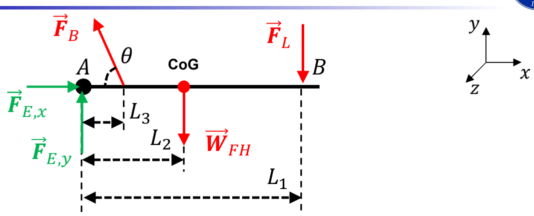


Step 2-b: where are forces applied?



$L_1 \stackrel{\text{def}}{=}$ distance from A to the load
$L_2 \stackrel{\text{def}}{=}$ distance from A to the CoG of the forearm+hand
$L_3 \stackrel{\text{def}}{=}$ distance from A to the force $\vec{F}_B$

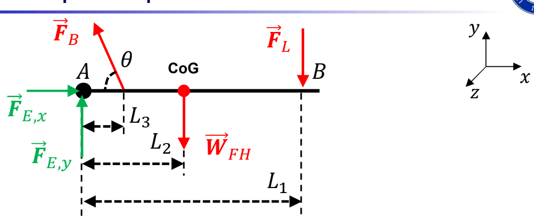
Step 3: equilibrium



x-axis: $F_{E,x} - F_B \cos\theta = 0$

y-axis: $F_{E,y} + F_B \sin\theta - W_{FH} - F_L = 0$

Step 3: equilibrium



x-axis: $F_{E,x} - F_B \cos\theta = 0$

y-axis: $F_{E,y} + F_B \sin\theta - W_{FH} - F_L = 0$

z-axis: $F_B L_3 \sin\theta - W_{FH} L_2 - F_L L_1 = 0$

Step 4: do the calculations

1) x-axis: $F_{E,x} - F_B \cos\theta = 0$

2) y-axis: $F_{E,y} + F_B \sin\theta - W_{FH} - F_L = 0$

3) z-axis: $F_B L_3 \sin\theta - W_{FH} L_2 - F_L L_1 = 0$

Step 4: do the calculations

1) x-axis: $F_{E,x} - F_B \cos\theta = 0$

2) y-axis: $F_{E,y} + F_B \sin\theta - W_{FH} - F_L = 0$

3) z-axis: $F_B L_3 \sin\theta - W_{FH} L_2 - F_L L_1 = 0$

Given:

$\theta = 75^{\circ}$
$F_L = 10$ lbf

Step 4: do the calculations

1) x-axis: $F_{E,x} - F_B \cos\theta = 0$

2) y-axis: $F_{E,y} + F_B \sin\theta - W_{FH} - F_L = 0$

3) z-axis: $F_B L_3 \sin\theta - W_{FH} L_2 - F_L L_1 = 0$

Given:

$\theta = 75^{\circ}$
$F_L = 10$ lbf

from eq. (1-3)

$F_{E,x} - F_B \cos(75°) = 0$

$F_{E,y} + F_B \sin(75°) - W_{FH} - 10 = 0$

$F_B L_3 \sin(75°) - W_{FH} L_2 - 10\, L_1 = 0$

What else can we get?

1) $F_{E,x} - \cos(75°)\ F_B = 0$

2) $F_{E,y} + \sin(75°)\, F_B - W_{FH} - 10 = 0$

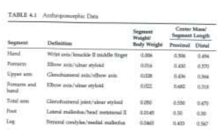
3) $\sin(75°)\, F_B L_3 - W_{FH} L_2 - 10 L_1 = 0$

What else can we get?

1) $F_{E,x} - \cos(75°)\ F_B = 0$

2) $F_{E,y} + \sin(75°)\, F_B - W_{FH} - 10 = 0$

3) $\sin(75°)\, F_B L_3 - W_{FH} L_2 - 10 L_1 = 0$



anthropometric tables

We can obtain from the table W_{FH}, L_2, and L_1 if we know **weight** and **height** of the person

What else can we get?

TABLE 4.1 Anthropomorphic Data

Segment	Definition	Segment Weight/ Body Weight	Center Mass/ Segment Length: Proximal	Center Mass/ Segment Length: Distal
Hand	Wrist axis/knuckle II middle finger	0.006	0.506	0.494
Forearm	Elbow axis/ulnar styloid	0.016	0.430	0.570
Upper arm	Glenohumeral axis/elbow axis	0.028	0.436	0.564
Forearm and hand	Elbow axis/ulnar styloid	0.022	0.682	0.318
Total arm	Glenohumeral joint/ulnar styloid	0.050	0.530	0.470
Foot	Lateral malleolus/head metatarsal II	0.0145	0.50	0.50
Leg	Femoral condyles/medial malleolus	0.0465	0.433	0.567

What else can we get?

TABLE 4.1 Anthropomorphic Data

Segment	Definition	Segment Weight/ Body Weight	Center Mass/ Segment Length Proximal	Center Mass/ Segment Length Distal
Hand	Wrist axis/knuckle II middle finger	0.006	0.506	0.494
Forearm	Elbow axis/ulnar styloid	0.016	0.430	0.570
Upper arm	Glenohumeral axis/elbow axis	0.028	0.436	0.564
Forearm and hand	Elbow axis/ulnar styloid	0.022	0.682	0.318
Total arm	Glenohumeral joint/ulnar styloid	0.050	0.530	0.470
Foot	Lateral malleolus/head metatarsal II	0.0145	0.50	0.50
Leg	Femoral condyles/medial malleolus	0.0465	0.433	0.567

weight = 160 lbf
height = 63 in

What else can we get?

TABLE 4.1 Anthropomorphic Data

Segment	Definition	Segment Weight/ Body Weight	Center Mass/ Segment Length Proximal	Center Mass/ Segment Length Distal
Hand	Wrist axis/knuckle II middle finger	0.006	0.506	0.494
Forearm	Elbow axis/ulnar styloid	0.016	0.430	0.570
Upper arm	Glenohumeral axis/elbow axis	0.028	0.436	0.564
Forearm and hand	Elbow axis/ulnar styloid	0.022	0.682	0.318
Total arm	Glenohumeral joint/ulnar styloid	0.050	0.530	0.470
Foot	Lateral malleolus/head metatarsal II	0.0145	0.50	0.50
Leg	Femoral condyles/medial malleolus	0.0465	0.433	0.567

weight = 160 lbf
height = 63 in

$$W_{FH} = (0.006 + 0.016) \times 160 = 3.52 \text{ lbf}$$

What else can we get?

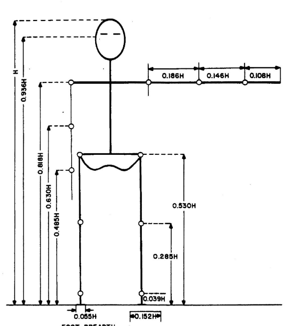


Figure 3.9 Some body segment lengths expressed as proportion of body stature by Drillis and Contini (Roebuck, Kroemer, and Thomson, 1975).

What else can we get?

weight = 160 lbf
height = 63 in

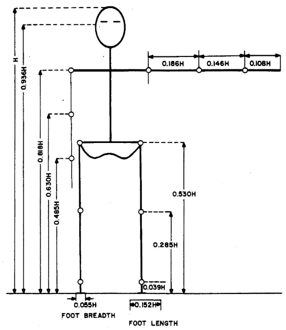
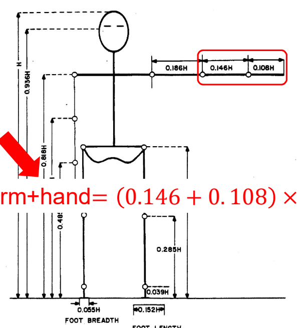


Figure 3.9 Some body segment lengths expressed as proportion of body stature by Drillis and Contini (Roebuck, Kroemer, and Thomson, 1975).

What else can we get?

weight = 160 lbf
height = 63 in

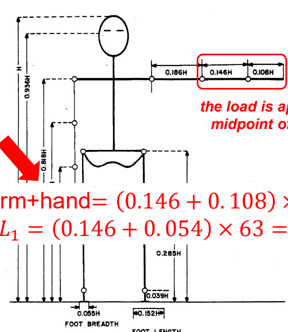


forearm+hand= $(0.146 + 0.108) \times 63 = 16$ in

Figure 3.9 Some body segment lengths expressed as proportion of body stature by Drillis and Contini (Roebuck, Kroemer, and Thomson, 1975).

What else can we get?

weight = 160 lbf
height = 63 in



the load is applied at the midpoint of the hand!

forearm+hand= $(0.146 + 0.108) \times 63 = 16$ in
$L_1 = (0.146 + 0.054) \times 63 = 12.6$ in

Figure 3.9 Some body segment lengths expressed as proportion of body stature by Drillis and Contini (Roebuck, Kroemer, and Thomson, 1975).

What else can we get?

TABLE 4.1 Anthropomorphic Data

Segment	Definition	Segment Weight/ Body Weight	Center Mass/ Segment Length Proximal	Center Mass/ Segment Length Distal
Hand	Wrist axis/knuckle II middle finger	0.006	0.506	0.494
Forearm	Elbow axis/ulnar styloid	0.016	0.430	0.570
Upper arm	Glenohumeral axis/elbow axis	0.028	0.436	0.564
Forearm and hand	Elbow axis/ulnar styloid	0.022	$\boxed{0.682}$	0.318
Total arm	Glenohumeral joint/ulnar styloid	0.050		
Foot	Lateral malleolus/head metatarsal II	0.0145		
Leg	Femoral condyles/medial malleolus	0.0465	0.433	0.567

we measure from the elbow!

weight=160 lbf
height = 63 in
forearm+hand =16 in

$$L_2 = 0.682 \times 16 = 10.91 \text{ in}$$

Step 4: do the calculations

1) $F_{E,x} - \cos(75°)\ F_B = 0$

2) $F_{E,y} + \sin(75°)\, F_B - W_{FH} - 10 = 0$

3) $\sin(75°)\, F_B L_3 - W_{FH} L_2 - 10 L_1 = 0$

Step 4: do the calculations

1) $F_{E,x} - \cos(75°)\ F_B = 0$

2) $F_{E,y} + \sin(75°)\, F_B - 3.52 - 10 = 0$

3) $\sin(75°)\, F_B L_3 - 3.52 \times 10.91 - 126 = 0$

Step 4: do the calculations

1) $F_{E,x} - \cos(75°)\ F_B = 0$

2) $F_{E,y} + \sin(75°)\ F_B - 3.52 - 10 = 0$

3) $\sin(75°)\ F_B L_3 - 3.52 \times 10.91 - 126 = 0$

L_3 is at the insertion point of the bicep in the forearm

Step 4: do the calculations

1) $F_{E,x} - \cos(75°)\ F_B = 0$

2) $F_{E,y} + \sin(75°)\ F_B - 3.52 - 10 = 0$

3) $\sin(75°)\ F_B L_3 - 3.52 \times 10.91 - 126 = 0$

L_3 is at the insertion point of the bicep in the forearm ➡ Let us assume: $L_3 = 2$ in (or look at a table!)

Step 4: do the calculations

1) $F_{E,x} - \cos(75°)\ F_B = 0$

2) $F_{E,y} + \sin(75°)\ F_B - 3.52 - 10 = 0$

3) $\sin(75°)\ F_B \times 2 - 3.52 \times 10.91 - 126 = 0$

Step 4: do the calculations

1) $F_{E,x} - \cos(75°)\ F_B = 0$

2) $F_{E,y} + \sin(75°)\ F_B - 3.52 - 10 = 0$

3) $\sin(75°)\ F_B \times 2 - 3.52 \times 10.91 - 126 = 0$

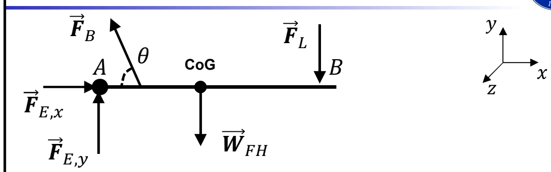
$\Downarrow$

$F_B = 85.1$ lbf

$F_{E,x} = 22.0$ lbf

$F_{E,y} = -68.7$ lbf

Step 5: fix the forces' direction

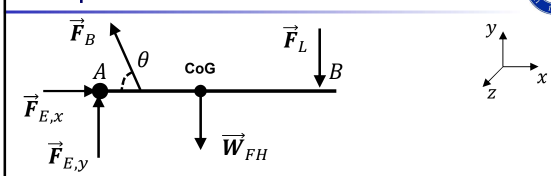


$F_B = 85.1$ lbf

$F_{E,x} = 22.0$ lbf

$F_{E,y} = -68.7$ lbf

Step 5: fix the forces' direction

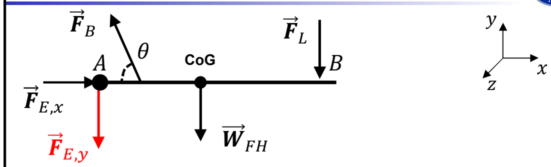


$F_B = 85.1$ lbf

$F_{E,x} = 22.0$ lbf

$F_{E,y} = -68.7$ lbf

Step 5: fix the forces' direction



$F_B = 85.1$ lbf

$F_{E,x} = 22.0$ lbf

$F_{E,y} = 68.7$ lbf