UCONN
UNIVERSITY OF CONNECTICUT

Circuit Analysis
Part II

ENGR 1166 Biomedical Engineering

Recap

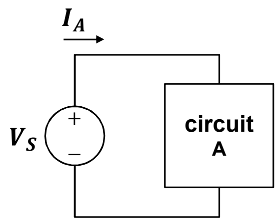
- **KCL:** At any node in an electrical circuit, the algebraic sum of the currents is equal to zero
- **KVL:** the directed sum of voltages along any closed path in an electrical circuit is zero
- **Ohm's law:** the ratio between the voltage drop at the terminals of an ideal resistor and the current passing through it is constant and is called "resistance"

Recap

- **Equivalent resistance:** Any circuit of resistors can be replaced with a single equivalent resistance
- **VDR:** A rule to compute the voltage drop across each resistor in a series of resistors
- **CDR:** A rule to compute the current through each resistor in a parallel circuit

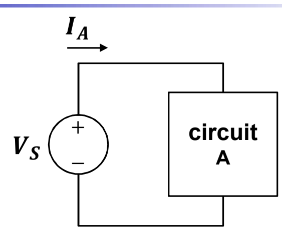
These tools are introduced for circuits made of resistors only!

Analysis of a more general circuit



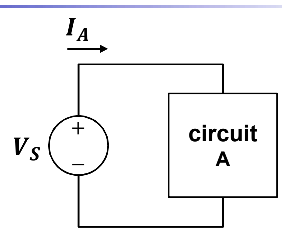
How to solve a circuit problem (e.g., compute unknown voltages and currents) when the circuit includes elements other than resistors?

Analysis of a more general circuit



The **node-voltage method** is a technique to **systematically** solve a circuit problem when elements other than resistors are involved

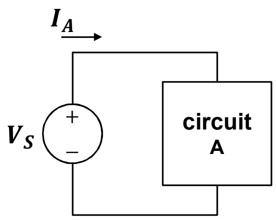
Our goal



If there are N nodes in the circuit, we want to determine up to $N-1$ equations with $N-1$ unknown variables and then solve

Node-voltage method

1) Set one **essential** node of circuit A as a reference (**ground**) and refer the voltage across any element in circuit A to it



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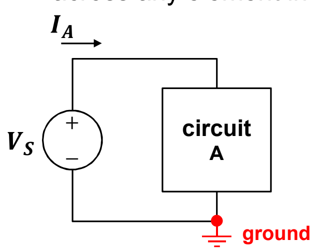

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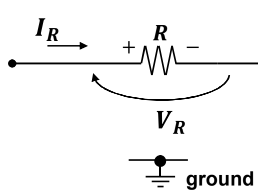

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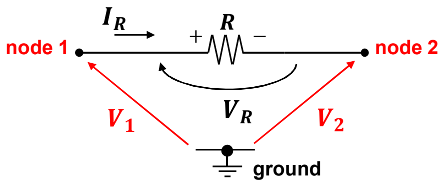

$V_R = V_1 - V_2$

Node-voltage method

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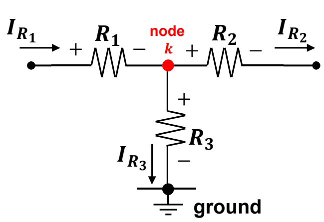

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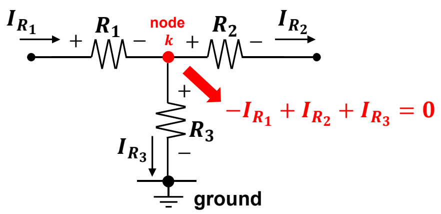

Node-voltage method

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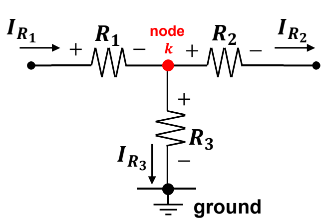

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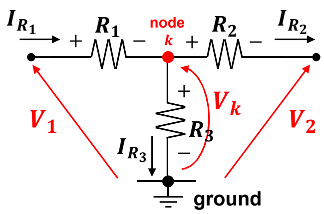

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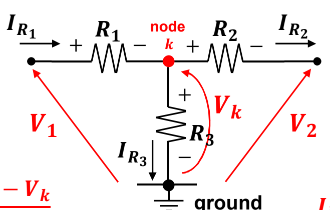

$$I_{R_3} = \frac{V_k}{R_3}$$

$$I_{R_1} = \frac{V_1 - V_k}{R_1}$$

$$I_{R_2} = \frac{V_k - V_2}{R_2}$$

Node-voltage method

4) Replace the voltages in the KCL equations

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$$-\frac{V_1 - V_k}{R_1} + \frac{V_k - V_2}{R_2} + \frac{V_k}{R_3} = 0$$

Node-voltage method

4) Replace the voltages in the KCL equations

$$-I_{R_1} + I_{R_2} + I_{R_3} = 0$$

$$-\frac{V_1 - V_k}{R_1} + \frac{V_k - V_2}{R_2} + \frac{V_k}{R_3} = 0$$

$$V_k = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \cdot \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

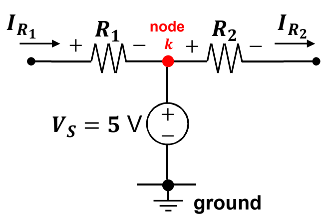
Node-voltage method

NOTE: I use KCL at the nodes to set the equations and I use Ohm's law to introduce voltages in these equations

I will solve with respect to voltages!

Node-voltage method

NOTE: If there is a voltage source between a node and the ground, there is no need to use KCL at that node as the voltage is known



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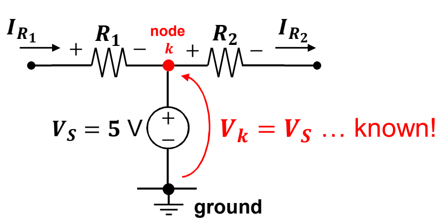
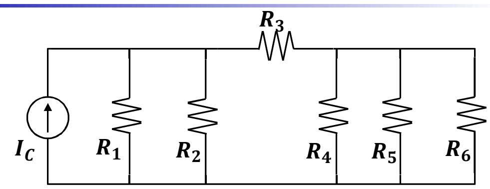


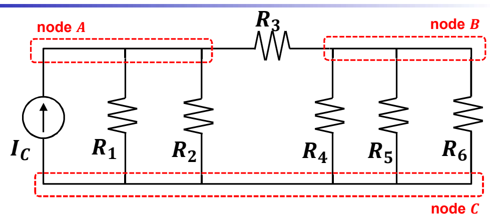
Example 1



$R_1 = 1\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $R_4 = 12\ \Omega$; $R_5 = 12\ \Omega$;
$R_6 = 12\ \Omega$; $I_C = 5\text{A}$

What is the voltage at each circuit element?

Example 1: where are the nodes?



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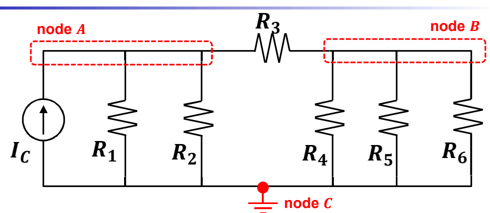
Example 1: set the ground



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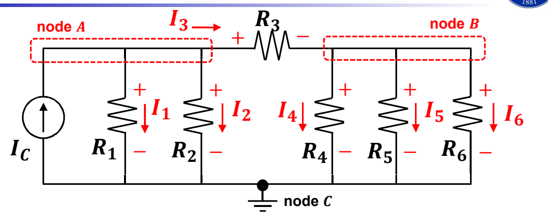
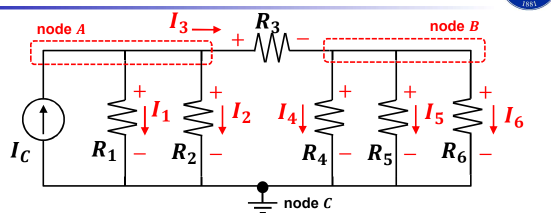
Example 1: set the current flow



$R_1 = 1\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $R_4 = 12\ \Omega$; $R_5 = 12\ \Omega$; $R_6 = 12\ \Omega$; $I_C = 5$A

NOTE: if not specified by the problem, we guess the direction of the currents now and we will fix it later when we have the solution

Example 1: KCL



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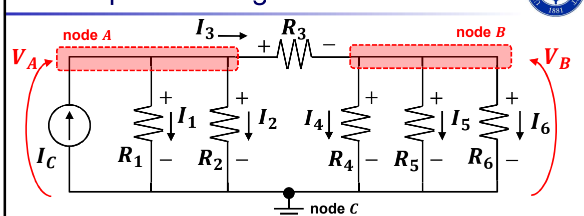
node A: $-I_C + I_1 + I_2 + I_3 = 0$

node B: $-I_3 + I_4 + I_5 + I_6 = 0$

Example 1: voltages



$R_1 = 1\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $R_4 = 12\ \Omega$; $R_5 = 12\ \Omega$; $R_6 = 12\ \Omega$; $I_C = 5$A

$I_1 = V_A/R_1$; $I_2 = V_A/R_2$; $I_3 = (V_A - V_B)/R_3$

$I_4 = V_B/R_4$; $I_5 = V_B/R_5$; $I_6 = V_B/R_6$

Example 1: voltages

node A: $-I_C + I_1 + I_2 + I_3 = 0$
node B: $-I_3 + I_4 + I_5 + I_6 = 0$

node A: $-I_C + \frac{V_A}{R_1} + \frac{V_A}{R_2} + \frac{V_A - V_B}{R_3} = 0$
node B: $-\frac{V_A - V_B}{R_3} + \frac{V_B}{R_4} + \frac{V_B}{R_5} + \frac{V_B}{R_6} = 0$

$R_1 = 1\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $R_4 = 12\ \Omega$; $R_5 = 12\ \Omega$; $R_6 = 12\ \Omega$; $I_C = 5$A

Example 1: solve!

node A: $-I_C + I_1 + I_2 + I_3 = 0$
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node A: $\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_A - \frac{1}{R_3}V_B = I_C$
node B: $\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}\right)V_B - \frac{1}{R_3}V_A = 0$

$R_1 = 1\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $R_4 = 12\ \Omega$; $R_5 = 12\ \Omega$; $R_6 = 12\ \Omega$; $I_C = 5$A

Example 1: solve!

node A: $-I_C + I_1 + I_2 + I_3 = 0$
node B: $-I_3 + I_4 + I_5 + I_6 = 0$

$$\frac{5}{2}V_A - V_B = 5$$
$$\frac{5}{4}V_B - V_A = 0$$

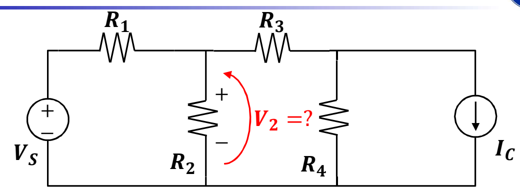
$R_1 = 1\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $R_4 = 12\ \Omega$; $R_5 = 12\ \Omega$; $R_6 = 12\ \Omega$; $I_C = 5$A

Example 1: solve!

node A: $-I_C + I_1 + I_2 + I_3 = 0$
node B: $-I_3 + I_4 + I_5 + I_6 = 0$

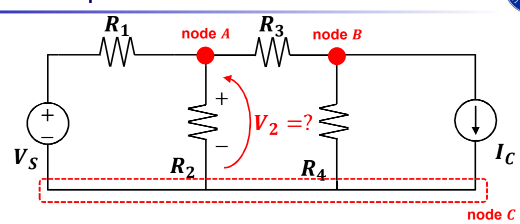
$$V_B \cong 2.35 \text{ V}$$
$$V_A \cong 2.94 \text{ V}$$

Example 2



$R_1 = 1/2\ \Omega$; $R_2 = 1/3\ \Omega$; $R_3 = 1/2\ \Omega$; $R_4 = 1/4\ \Omega$;
$V_S = 5$ V; $I_C = 3$ A

Example 2: where are the nodes?



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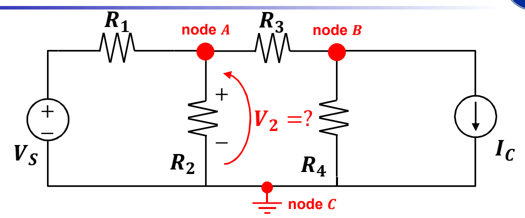
Example 2: set the ground



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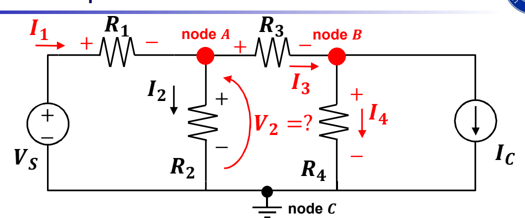
Example 2: set the current flow



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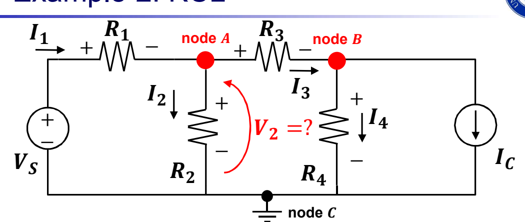
NOTE: The direction of I_2 and I_C is set by the problem while we need to guess the direction of the remaining currents

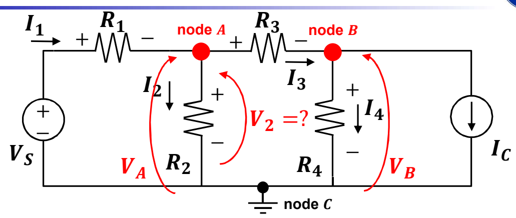
Example 2: KCL



$R_1 = 1/2\ \Omega$; $R_2 = 1/3\ \Omega$; $R_3 = 1/2\ \Omega$; $R_4 = 1/4\ \Omega$;
$V_S = 5$ V; $I_C = 3$ A

node A: $-I_1 + I_2 + I_3 = 0$
node B: $-I_3 + I_4 + I_C = 0$

Example 2: voltages



$R_1 = 1/2\ \Omega$; $R_2 = 1/3\ \Omega$; $R_3 = 1/2\ \Omega$; $R_4 = 1/4\ \Omega$;
$V_S = 5$ V; $I_C = 3$ A

$$I_1 = (V_S - V_A)/R_1\,;\ I_2 = V_A/R_2\,;\ V_2 = V_A$$
$$I_3 = (V_A - V_B)/R_3\,;\ I_4 = V_B/R_4$$

Example 2: voltages

node A: $-I_1 + I_2 + I_3 = 0$
node B: $-I_3 + I_4 + I_C = 0$

node A: $-\frac{V_S - V_A}{R_1} + \frac{V_A}{R_2} + \frac{V_A - V_B}{R_3} = 0$
node B: $-\frac{V_A - V_B}{R_3} + \frac{V_B}{R_4} + I_C = 0$

$R_1 = 1/2\ \Omega$; $R_2 = 1/3\ \Omega$; $R_3 = 1/2\ \Omega$; $R_4 = 1/4\ \Omega$;
$V_S = 5$ V; $I_C = 3$ A

Example 2: solve!

node A: $-I_1 + I_2 + I_3 = 0$
node B: $-I_3 + I_4 + I_C = 0$

node A: $\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_A - \frac{1}{R_3} V_B - \frac{1}{R_1} V_S = 0$
node B: $\left(\frac{1}{R_3} + \frac{1}{R_4}\right) V_B - \frac{1}{R_3} V_A + I_C = 0$

$R_1 = 1/2\ \Omega$; $R_2 = 1/3\ \Omega$; $R_3 = 1/2\ \Omega$; $R_4 = 1/4\ \Omega$;
$V_S = 5$ V; $I_C = 3$ A

Example 2: solve!

node A: $-I_1 + I_2 + I_3 = 0$
node B: $-I_3 + I_4 + I_C = 0$

$$7V_A - 2V_B - 10 = 0$$
$$6V_B - 2V_A + 3 = 0$$

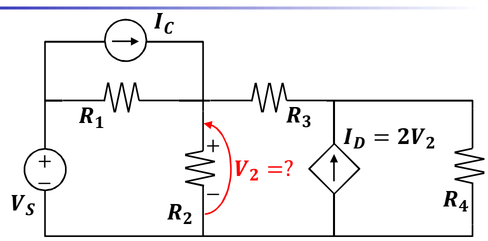
Example 2: solve!

node A: $-I_1 + I_2 + I_3 = 0$
node B: $-I_3 + I_4 + I_C = 0$

$$V_A = 3V_B + 3/2$$
$$21V_B + 21/2 - 2V_B - 10 = 0$$

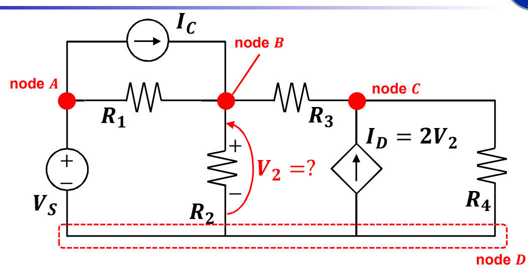
$V_A = \frac{27}{19} \cong 1.42$ V
$V_B = -\frac{1}{38} \cong -0.03$ V

Example 3



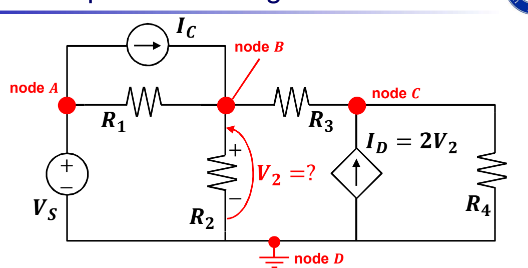
$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

Example 3: where are the nodes?



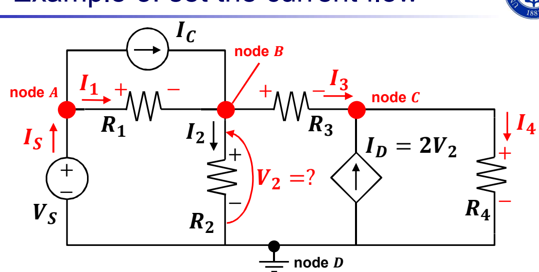
$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

Example 3: set the ground



$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

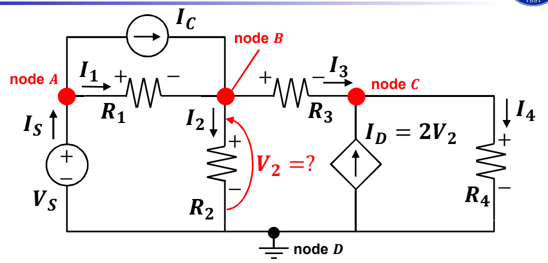
Example 3: set the current flow



$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

NOTE: The direction of I_2, I_C, and I_D is set by the problem

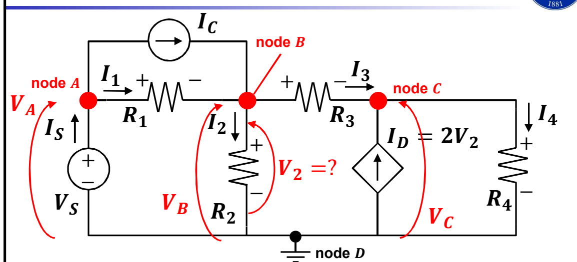
Example 3: KCL



$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

node A: $-I_S + I_C + I_1 = 0$
node B: $-I_1 - I_C + I_2 + I_3 = 0$
node C: $-I_3 - I_D + I_4 = 0$

Example 3: voltages



$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

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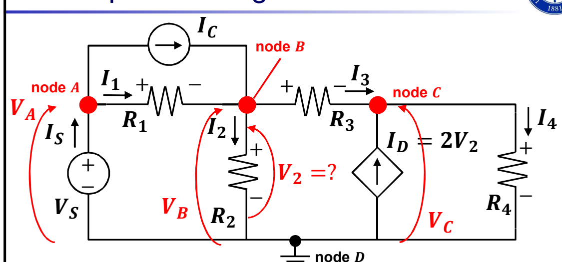
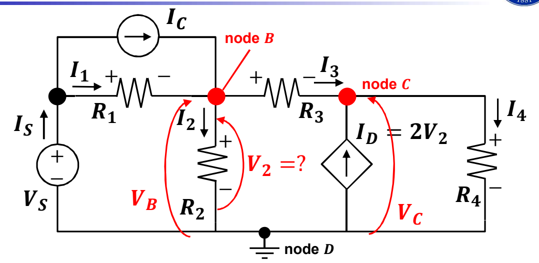


$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

NOTE: $V_A = V_S = 3$ V… we do not need KCL at node A!

Example 3: voltages



$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

$$I_1 = (V_S - V_B)/R_1\ ;\ I_2 = V_B/R_2\ ;\ V_2 = V_B$$

$$I_3 = (V_B - V_C)/R_3\ ;\ I_4 = V_C/R_4$$

Example 3: voltages

node B: $-I_1 - I_C + I_2 + I_3 = 0$

node C: $-I_3 - I_D + I_4 = 0$

node B: $-\frac{V_S - V_B}{R_1} - I_C + \frac{V_B}{R_2} + \frac{V_B - V_C}{R_3} = 0$

node C: $-\frac{V_B - V_C}{R_3} - 2V_B + \frac{V_C}{R_4} = 0$

$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

Example 3: solve!

node B: $-I_1 - I_C + I_2 + I_3 = 0$

node C: $-I_3 - I_D + I_4 = 0$

node B: $\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_B - \frac{1}{R_3}V_C - \frac{1}{R_1}V_S - I_C = 0$

node C: $\left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_C - \left(2 + \frac{1}{R_3}\right)V_B = 0$

$R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 4\ \Omega$; $R_4 = 5\ \Omega$; $V_S = 3$ V; $I_C = 2$ A

Example 3: solve!

node B: $-I_1 - I_C + I_2 + I_3 = 0$
node C: $-I_3 - I_D + I_4 = 0$

$$\frac{13}{12}V_B - \frac{1}{4}V_C - \frac{3}{2} - 2 = 0$$
$$\frac{9}{20}V_C - \frac{9}{4}V_B = 0$$

Example 3: solve!

node B: $-I_1 - I_C + I_2 + I_3 = 0$
node C: $-I_3 - I_D + I_4 = 0$

$$V_C = 5V_B$$
$$\frac{13}{12}V_B - \frac{5}{4}V_B = \frac{7}{2}$$

$$V_B = -21 \text{ V}$$
$$V_C = -105 \text{ V}$$