UCONN
UNIVERSITY OF CONNECTICUT

Biomechanical Modeling and Analysis

ENGR 1166 Biomedical Engineering

Recap

A rigid body is **in equilibrium** if (1) the resultant of all the forces acting on the body is zero and (2) the resultant of all the moments about a given point A on the body is zero

$$\sum_i \vec{\boldsymbol{F}}_i = \mathbf{0}; \quad \sum_i \vec{\boldsymbol{\tau}}_{i,A} = \mathbf{0}$$

Recap

- A free body diagram (FBD) is a pictorial device to analyze the forces and moments acting on a body
- The body may consist of many components, each one acting as a single body. If so, a whole series of FBDs may be necessary
- In a FBD, constraints are replaced by arrows representing the forces and moments they generate

Today we focus on…

- A free body diagram (FBD) is a pictorial device to analyze the forces and moments acting on a body
- The body may consist of many components, each one acting as a single body. If so, **a whole series of FBDs may be necessary**
- In a FBD, constraints are replaced by arrows representing the forces and moments they generate

An example: the human arm

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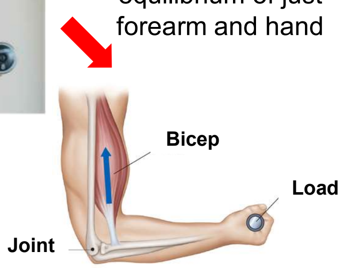
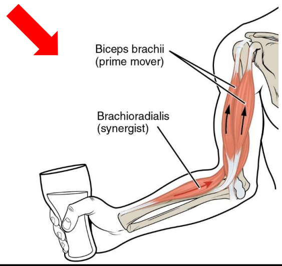
In case study I we focused on the equilibrium of just forearm and hand

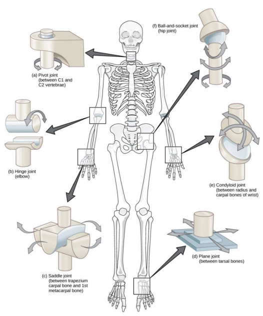


An example: the human arm

What if we consider the effects of the forces at the shoulder?

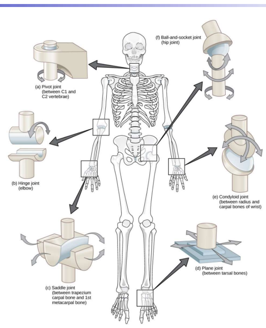


Human body has many segments



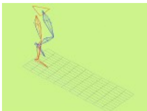
Depending on the application, each part between two joints may be considered a segment to be analyzed

Segmental analysis



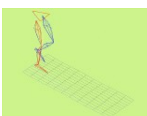
The ultimate goal is to know what the muscles and joint are doing when a movement is performed

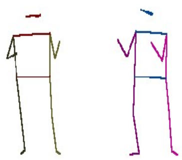
Segmental analysis: assumptions



- The human skeleton is represented as a series of links
- Many link systems have been used (e.g., cones, spheres, etc.)

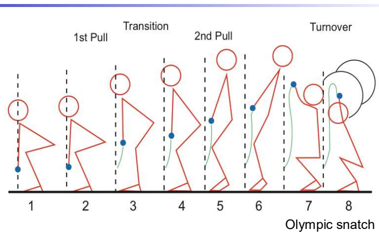
Segmental analysis: assumptions





- The human skeleton is represented as a series of links
- Many link systems have been used (e.g., cones, spheres, etc.)
- The simplest and most adopted link system is the **stick man**

The stick man model



- The stick man uses straight sticks to represent each of the links
- Remember, each segment is a rigid body connected at joints!
- Remember, each segment has a CoG and a weight!

Segmental analysis: a case study

We want to study the effects of position and load on the lower back and understand under what conditions the subject can be hurt

Key points of the segmental analysis

With regard to the case study, we will follow these rules:

1) Break the body into rigid body segments (stick man representation)

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With regard to the case study, we will follow these rules:

1) Break the body into rigid body segments (stick man representation)
2) Analyze each segment using a FBD
3) At the intersection of two segments, the forces in the first segment are equal in magnitude and opposite in direction to the forces in the adjoining segment

Key points of the segmental analysis

With regard to the case study, we will follow these rules:

4) At the intersection of two segments, the moment in the first segment also occurs in the adjoining segment

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4) At the intersection of two segments, the moment in the first segment also occurs in the adjoining segment
5) Reduce the number of equations by using substitution
6) Solve the equations by replacing variables with quantitative values

Step 1: what assumptions?

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- ❑ The system is **in equilibrium**
- ❑ The analysis is in 2D
- ❑ We want to study the load at the base of the trunk

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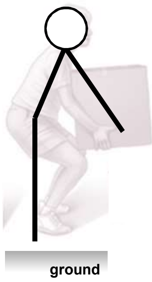

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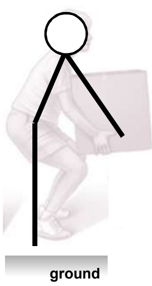

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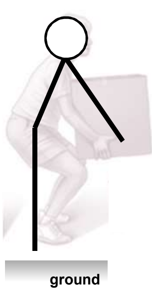
- We can subsume all in three rigid bodies: (1) arm segment; (2) trunk segment; (3) leg segment
- Segments (2-3) must account for two arms and two legs!

Step 1: what else?



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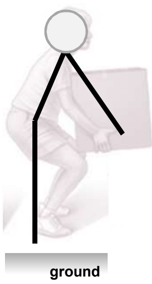

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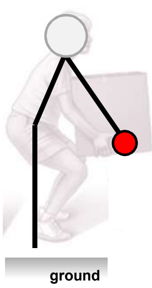

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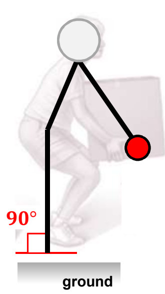

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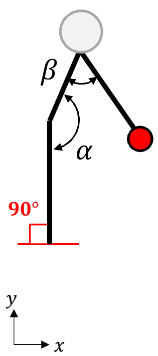
Step 2: derive the FBDs



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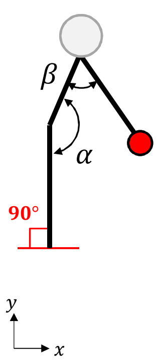
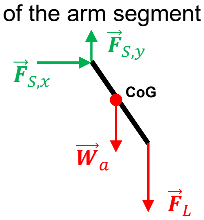
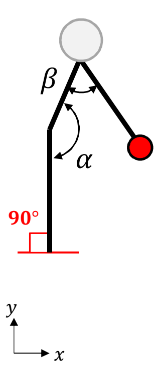


A) FBD of the arm segment

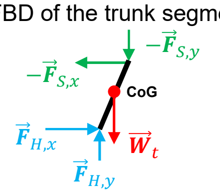


$\vec{F}_L \stackrel{\text{def}}{=}$ weight of the load
$\vec{W}_a \stackrel{\text{def}}{=}$ weight of one single arm segment
$\vec{F}_{S,x}, \vec{F}_{S,y} \stackrel{\text{def}}{=}$ forces (in the x and y direction) at the shoulder joint

Step 2: derive the FBDs

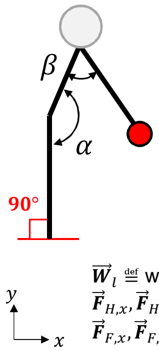


B) FBD of the trunk segment

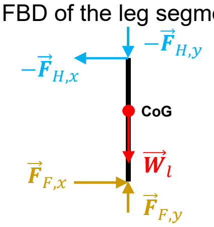


$\vec{W}_t \stackrel{\text{def}}{=}$ weight of the trunk segment
$\vec{F}_{S,x}, \vec{F}_{S,y} \stackrel{\text{def}}{=}$ forces (in the x and y direction) at the shoulder joint
$\vec{F}_{H,x}, \vec{F}_{H,y} \stackrel{\text{def}}{=}$ forces (in the x and y direction) at the hip joint

Step 2: derive the FBDs



C) FBD of the leg segment



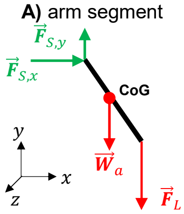
$\vec{W}_l \stackrel{\text{def}}{=}$ weight of one single leg segment

$\vec{F}_{H,x}, \vec{F}_{H,y} \stackrel{\text{def}}{=}$ forces (in the x and y direction) at the hip

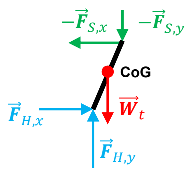
$\vec{F}_{F,x}, \vec{F}_{F,y} \stackrel{\text{def}}{=}$ forces (in the x and y direction) at the ankle

Step 3: equilibrium

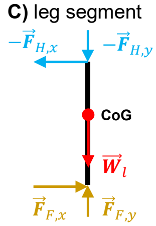
A) arm segment



B) trunk segment



C) leg segment



Step 3: equilibrium

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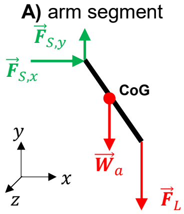

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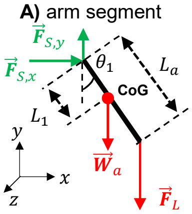

$L_a \stackrel{\text{def}}{=}$ length of the arm segment

$L_1 \stackrel{\text{def}}{=}$ distance from the shoulder to the CoG of the arm

Step 3: equilibrium

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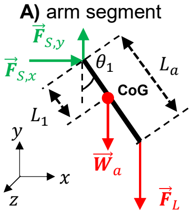

$L_a \stackrel{\text{def}}{=}$ length of the arm segment

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$x)\ F_{S,x} = 0$ $\quad$ $y)\ F_{S,y} - \mathbf{2}W_a - F_L = 0$

Step 3: equilibrium

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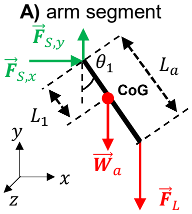

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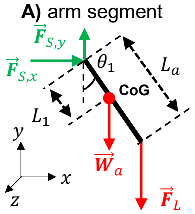
moments WRT the shoulder joint:

$$-F_L L_a \sin\theta_1 - \mathbf{2}W_a L_1 \sin\theta_1 + M_t = 0$$

Step 3: equilibrium

A) arm segment



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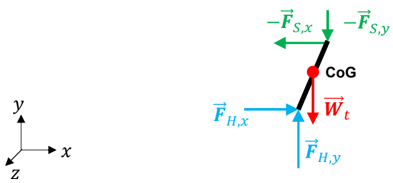
moment at the joint that the shoulder exerts to keep the arm in equilibrium

Step 3: equilibrium

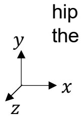
B) trunk segment



Step 3: equilibrium

$L_t \stackrel{\text{def}}{=}$ length of the trunk segment

$L_2 \stackrel{\text{def}}{=}$ distance from the hip to the CoG of the trunk



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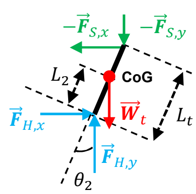

Step 3: equilibrium

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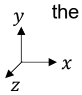

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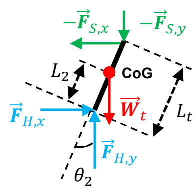
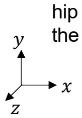


$x)\ -F_{S,x} + F_{H,x} = 0 \qquad y)\ -F_{S,y} + F_{H,y} - W_t = 0$

Step 3: equilibrium

$L_t \stackrel{\text{def}}{=}$ length of the trunk segment

$L_2 \stackrel{\text{def}}{=}$ distance from the hip to the CoG of the trunk



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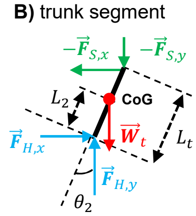


$x)\ -F_{S,x} + F_{H,x} = 0 \qquad y)\ -F_{S,y} + F_{H,y} - W_t = 0$

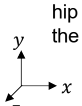
moments WRT the hip joint:

$$-F_{S,y} L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

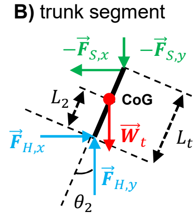
Step 3: equilibrium

$L_t \stackrel{\text{def}}{=}$ length of the trunk segment

$L_2 \stackrel{\text{def}}{=}$ distance from the hip to the CoG of the trunk



B) trunk segment



$x)\ -F_{S,x} + F_{H,x} = 0 \qquad y)\ -F_{S,y} + F_{H,y} - W_t = 0$

moments WRT the hip joint:

$$-F_{S,y} L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

it comes from the FBD of the previous segment

Step 3: equilibrium

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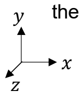
$L_2 \stackrel{\text{def}}{=}$ distance from the hip to the CoG of the trunk



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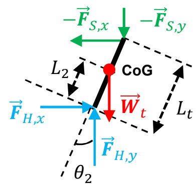

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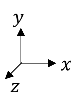
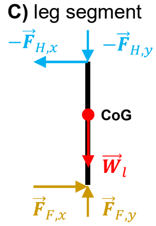
moment at the joint that is exerted to keep the trunk in equilibrium

Step 3: equilibrium

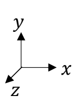


C) leg segment



Step 3: equilibrium

$L_l \stackrel{\text{def}}{=}$ length of the leg segment



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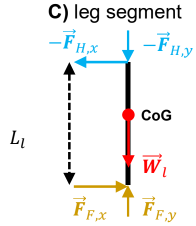

Step 3: equilibrium

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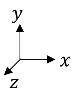
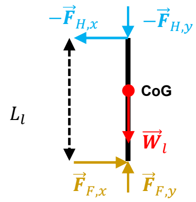
C) leg segment





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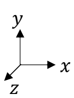
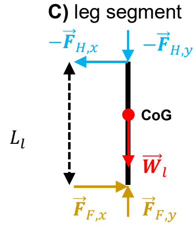
Step 3: equilibrium

$L_l \overset{\text{def}}{=}$ length of the leg segment

C) leg segment





$x)\ -F_{H,x} + F_{F,x} = 0 \quad y)\ -F_{H,y} + F_{F,y} - 2W_l = 0$

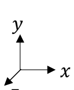
moments WRT the ankle joint:

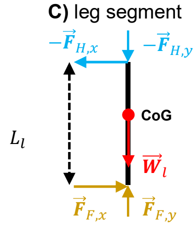
$$F_{H,x}L_l - M_l + M_f = 0$$

Step 3: equilibrium

$L_l \overset{\text{def}}{=}$ length of the leg segment

C) leg segment





$x)\ -F_{H,x} + F_{F,x} = 0 \quad y)\ -F_{H,y} + F_{F,y} - 2W_l = 0$

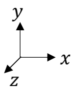
moments WRT the ankle joint:

$$F_{H,x}L_l - M_l + M_f = 0$$

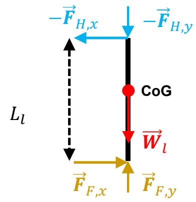
from the FBD of the previous segment

Step 3: equilibrium

$L_l \stackrel{\text{def}}{=}$ length of the leg segment



C) leg segment



$$x)\ -F_{H,x} + F_{F,x} = 0 \quad y)\ -F_{H,y} + F_{F,y} - 2W_l = 0$$

moments WRT the ankle joint:

$$F_{H,x}L_l - M_l + M_f = 0$$

due to the interaction with the foot segment and the ground

What we have so far

A) arm segment

$$F_{S,x} = 0$$
$$F_{S,y} - 2W_a - F_L = 0$$
$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$

B) trunk segment

$$-F_{S,x} + F_{H,x} = 0$$
$$-F_{S,y} + F_{H,y} - W_t = 0$$
$$-F_{S,y} L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

C) leg segment

$$-F_{H,x} + F_{F,x} = 0$$
$$-F_{H,y} + F_{F,y} - 2W_l = 0$$
$$F_{H,x}L_l - M_l + M_f = 0$$

What we have so far

A) arm segment

$$F_{S,x} = 0$$
$$F_{S,y} - 2W_a - F_L = 0$$
$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$

B) trunk segment

$$F_{H,x} = F_{S,x} = 0$$
$$-F_{S,y} + F_{H,y} - W_t = 0$$
$$-F_{S,y} L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

C) leg segment

$$F_{F,x} = F_{H,x} = 0$$
$$-F_{H,y} + F_{F,y} - 2W_l = 0$$
$$\cancel{F_{H,x}L_l} - M_l + M_f = 0$$

What we have so far

$$F_{S,y} - 2W_a - F_L = 0$$
$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-F_{S,y} + F_{H,y} - W_t = 0$$
$$-F_{S,y} L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$
$$-F_{H,y} + F_{F,y} - 2W_l = 0$$

$M_f = M_l$

What we have so far

$$F_{S,y} - 2W_a - F_L = 0$$
$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-F_{S,y} + F_{H,y} - W_t = 0$$
$$-F_{S,y} L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$
$$-F_{H,y} + F_{F,y} - 2W_l = 0$$

$M_f = M_l$

Variables to solve for:

$F_{S,y}$ M_t $F_{H,y}$ M_l $F_{F,y}$

Reducing the number of equations

$$F_{S,y} - 2W_a - F_L = 0$$
$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-F_{S,y} + F_{H,y} - W_t = 0$$
$$-F_{S,y} L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$
$$-F_{H,y} + F_{F,y} - 2W_l = 0$$

$M_f = M_l$

Reducing the number of equations

$$F_{S,y} - 2W_a - F_L = 0$$
$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-F_{S,y} + F_{H,y} - W_t = 0$$
$$-F_{S,y} L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$
$$-F_{H,y} + F_{F,y} - 2W_l = 0$$

$M_f = M_l$

Reducing the number of equations

$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-(2W_a + F_L) + F_{H,y} - W_t = 0$$
$$-(2W_a + F_L) L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$
$$-F_{H,y} + F_{F,y} - 2W_l = 0$$

$M_f = M_l$
$F_{S,y} = 2W_a + F_L$

Reducing the number of equations

$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-(2W_a + F_L) + F_{H,y} - W_t = 0$$
$$-(2W_a + F_L) L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$
$$-F_{H,y} + F_{F,y} - 2W_l = 0$$

$M_f = M_l$
$F_{S,y} = 2W_a + F_L$

Reducing the number of equations

$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-(2W_a + F_L) + F_{H,y} - W_t = 0$$
$$-(2W_a + F_L) L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

$M_f = M_l$

$F_{S,y} = 2W_a + F_L$

$F_{F,y} = F_{H,y} + 2W_l$

Step 4: do the calculations

$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-(2W_a + F_L) + F_{H,y} - W_t = 0$$
$$-(2W_a + F_L) L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

$M_f = M_l$

$F_{S,y} = 2W_a + F_L$

$F_{F,y} = F_{H,y} + 2W_l$

Given:

F_L =10 lbf
subject's weight = 160 lbf
subject's height = 72 in

Step 4: do the calculations

$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-(2W_a + F_L) + F_{H,y} - W_t = 0$$
$$-(2W_a + F_L) L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

$M_f = M_l$

$F_{S,y} = 2W_a + F_L$

$F_{F,y} = F_{H,y} + 2W_l$

Given:

F_L =10 lbf
subject's weight = 160 lbf
subject's height = 72 in

from tables

$W_a = 0.050 \times 160 = 8.0$ lbf
$W_t = 0.497 \times 160 = 79.5$ lbf
$W_l = 0.0465 \times 160 = 7.4$ lbf

Step 4: do the calculations

$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-(2W_a + F_L) + F_{H,y} - W_t = 0$$
$$-(2W_a + F_L)L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

$M_f = M_l$

$F_{S,y} = 2W_a + F_L$

$F_{F,y} = F_{H,y} + 2W_l$

Given:

F_L =10 lbf
subject's weight = 160 lbf
subject's height = 72 in

from tables

$W_a = 0.050 \times 160 = 8.0$ lbf
$W_t = 0.497 \times 160 = 79.5$ lbf
$W_l = 0.0465 \times 160 = 7.4$ lbf

$L_a = 0.440 \times 72 = 31.7$ in
$L_t = 0.288 \times 72 = 20.7$ in

Step 4: do the calculations

$$-F_L L_a \sin\theta_1 - 2W_a L_1 \sin\theta_1 + M_t = 0$$
$$-(2W_a + F_L) + F_{H,y} - W_t = 0$$
$$-(2W_a + F_L)L_t \sin\theta_2 - W_t L_2 \sin\theta_2 - M_t + M_l = 0$$

$M_f = M_l$

$F_{S,y} = 2W_a + F_L$

$F_{F,y} = F_{H,y} + 2W_l$

Given:

F_L =10 lbf
subject's weight = 160 lbf
subject's height = 72 in

from tables

$W_a = 0.050 \times 160 = 8.0$ lbf
$W_t = 0.497 \times 160 = 79.5$ lbf
$W_l = 0.0465 \times 160 = 7.4$ lbf

$L_1 = 0.530 \times L_a = 16.8$ in
$L_2 = 0.500 \times L_t = 10.4$ in

Step 4: do the calculations

$$-10 \times 31.7 \sin\theta_1 - 2 \times 8 \times 16.8 \sin\theta_1 + M_t = 0$$
$$-(2 \times 8 + 10) + F_{H,y} - 79.5 = 0$$
$$-26 \times 20.7 \sin\theta_2 - 79.5 \times 10.4 \sin\theta_2 - M_t + M_l = 0$$

$M_f = M_l$

$F_{S,y} = 2 \times 8 + 10 = 26$

$F_{F,y} = F_{H,y} + 2 \times 7.4$

Step 4: do the calculations

$$-317\sin\theta_1 - 268.6\sin\theta_1 + M_t = 0$$

$$-538.2\sin\theta_2 - 824.3\sin\theta_2 - M_t + M_l = 0$$

$M_f = M_l$
$F_{S,y} = 26$ lbf
$F_{H,y} = 105.5$ lbf
$F_{F,y} = 120.3$ lbf

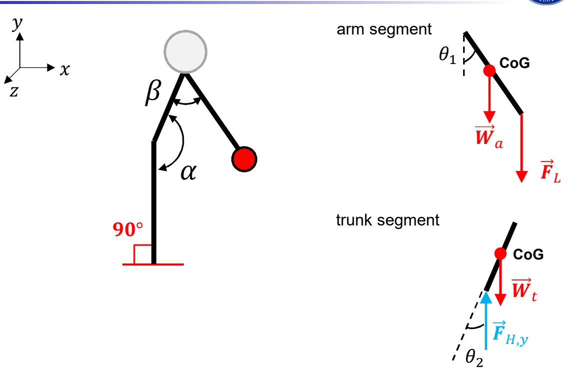
Parameters to be derived

$$-317\sin\theta_1 - 268.6\sin\theta_1 + M_t = 0$$

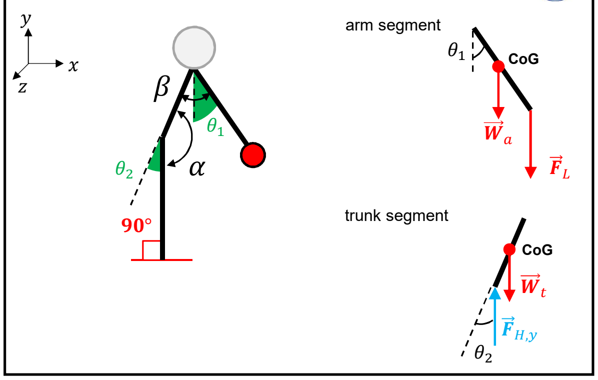
$$-538.2\sin\theta_2 - 824.3\sin\theta_2 - M_t + M_l = 0$$

$M_f = M_l$
$F_{S,y} = 26$ lbf
$F_{H,y} = 105.5$ lbf
$F_{F,y} = 120.3$ lbf

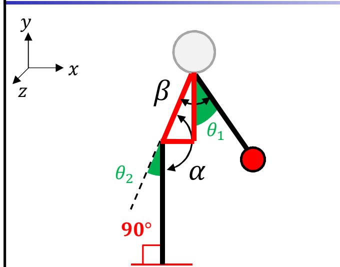
Parameters to be derived

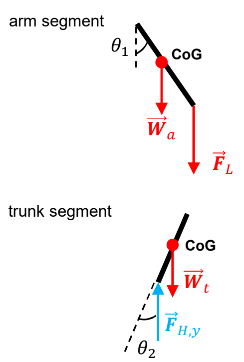


Parameters to be derived



Parameters to be derived





$\theta_1 = \alpha + \beta - 180° = 50°$
$\theta_2 = 180° - \alpha = 20°$

Solution to the segmental analysis

$$-317\sin(50°) - 268.6\sin(50°) + M_t = 0$$

$$-538.2\sin(20°) - 824.3\sin(20°) - M_t + M_l = 0$$

$M_f = M_l$
$F_{S,y} = 26$ lbf
$F_{H,y} = 105.5$ lbf
$F_{F,y} = 120.3$ lbf

Solution to the segmental analysis

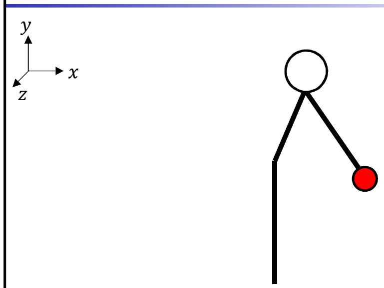
$$-317\sin(50°) - 268.6\sin(50°) + M_t = 0$$

$$-538.2\sin(20°) - 824.3\sin(20°) - M_t + M_l = 0$$

$M_f = M_l$
$F_{S,y} = 26$ lbf
$F_{H,y} = 105.5$ lbf
$F_{F,y} = 120.3$ lbf

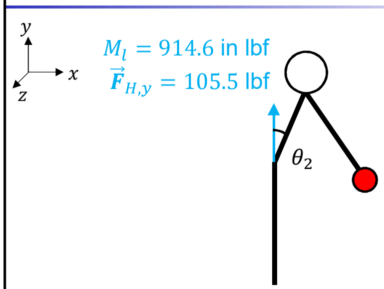
$M_t = 448.6$ in lbf
$M_l = 914.6$ in lbf

Conclusions

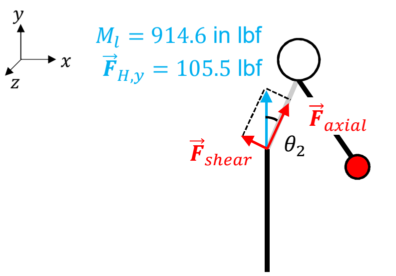


We wanted to know the effects of position and load on the lower back

Conclusions



Conclusions



axial (compressive) and shear forces act on the spine

$$F_{axial} = F_{H,y} \cos \theta_2$$

$$F_{shear} = F_{H,y} \sin \theta_2$$

Conclusions