Introduction to Bioelectricity
Part I

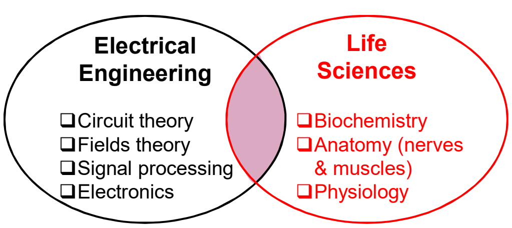
ENGR 1166 Biomedical Engineering

## What is "bioelectricity"?

- It studies the electric phenomena produced by or occurring within living organisms

## What is "bioelectricity"?

- It studies the electric phenomena produced by or occurring within living organisms
- It combines Engineering and Life Sciences



## Bioelectricity: areas of interest

Bioelectricity

## Bioelectricity: areas of interest

Bioelectricity

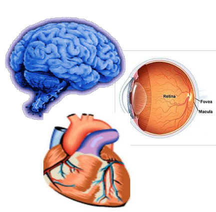
To measure the electric signals produced by the activity of living tissues

## Bioelectricity: areas of interest

Bioelectricity

To measure the electric signals produced by the activity of living tissues

- ❑ Electroencephalography (EEG)
- ❑ Eectroretinography (ERG)
- ❑ Electrocardiography (ECG)
- ❑ electromyography (EMG)



## Bioelectricity: areas of interest

Bioelectricity

- To measure the electric signals produced by the activity of living tissues
- To study the effect of electric fields due to an external device on tissue

## Bioelectricity: areas of interest

Bioelectricity

- To mea... signals...
- To study the effect of electric fields due to an external device on tissue

- ❑ Deep brain stimulation (DBS)
- ❑ Functional stimulation (FES)
- ❑ Cardiac defibrillation (CDF)

## Bioelectricity: key concepts

Electricity

*The set of physical phenomena associated with the presence and flow of electric charge*

## Bioelectricity: key concepts

Electricity

*The set of physical phenomena associated with the presence and flow of electric charge*

- Charge $\stackrel{\text{def}}{=}$ *A property of subatomic particles determining their electromagnetic interactions*

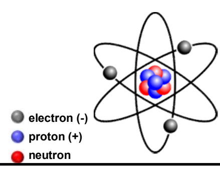
## Bioelectricity: key concepts

Electricity

*The set of physical phenomena associated with the presence and flow of electric charge*

- Charge $\stackrel{\text{def}}{=}$ *A property of subatomic particles determining their electromagnetic interactions*

Atoms have particles with positive (**protons**) and negative (**electrons**) charge




## Bioelectricity: key concepts

Electricity

*The set of physical phenomena associated with the presence and flow of electric charge*

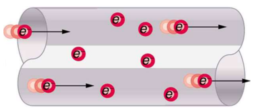
- Charge $\stackrel{\text{def}}{=}$ *A property of subatomic particles determining their electromagnetic interactions*
- Electric current $\stackrel{\text{def}}{=}$ *A movement or flow of electrically charged particles through a medium*

## Bioelectricity: key concepts

**Electricity**

*The set of physical phenomena associated with the presence and flow of electric charge*

- ❑Charge $\stackrel{\text{def}}{=}$ *A property of subatomic particles determining their electromagnetic interactions*
- ❑Electric current $\stackrel{\text{def}}{=}$ *A movement or flow of electrically charged particles through a medium*



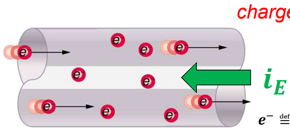
$e^-$ $\stackrel{\text{def}}{=}$ **electron**

## Bioelectricity: key concepts

**Electricity**

*The set of physical phenomena associated with the presence and flow of electric charge*

- ❑Charge $\stackrel{\text{def}}{=}$ *A property of subatomic particles determining their electromagnetic interactions*
- ❑Electric current $\stackrel{\text{def}}{=}$ *A movement or flow of electrically charged particles through a medium*




$e^-$ $\stackrel{\text{def}}{=}$ **electron**

By definition, the direction of the current $i_E$ is given by the direction of positive charges

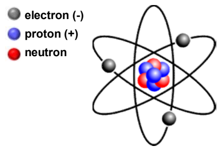
## Bioelectricity: key concepts

**Electricity**

*The set of physical phenomena associated with the presence and flow of electric charge*

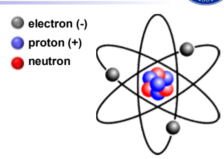
- ❑Charge $\stackrel{\text{def}}{=}$ *A property of subatomic particles determining their electromagnetic interactions*
- ❑Electric current $\stackrel{\text{def}}{=}$ *A movement or flow of electrically charged particles through a medium*
- ❑Voltage $\stackrel{\text{def}}{=}$ *The work required to move a unit charge between two points*

## Electric charge



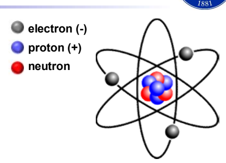

## Electric charge

It is **quantized**, i.e., it comes in integer multiples of an individual small unit called "**elementary charge**" ***e***



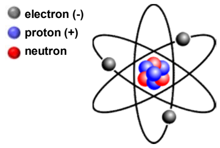

## Electric charge

It is **quantized**, i.e., it comes in integer multiples of an individual small unit called "**elementary charge**" ***e***




In the SI, it is measured in **Coulomb** (**C**)

## Electric charge



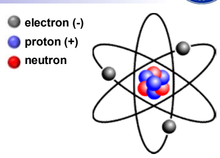

It is **quantized**, i.e., it comes in integer multiples of an individual small unit called "**elementary charge**" $\boldsymbol{e}$

In the SI, it is measured in **Coulomb** (**C**)

$\boldsymbol{e} \cong 1.602 \times 10^{-19}$ **C**

$1$ **C**$= \boldsymbol{e}/(1.602 \times 10^{-1}\ ) \cong 6.241 \times 10^{18}\boldsymbol{e}$

## Electric charge




It is **quantized**, i.e., it comes in integer multiples of an individual small unit called "**elementary charge**" $\boldsymbol{e}$

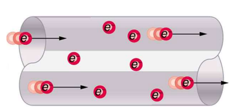
In the SI, it is measured in **Coulomb** (**C**)

$\boldsymbol{e} \cong 1.602 \times 10^{-19}$ **C**

$1$ **C**$= \boldsymbol{e}/(1.602 \times 10^{-19}) \cong 6.241 \times 10^{18}\boldsymbol{e}$

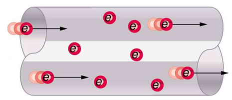
An electron and a proton have both the charge of 1 $\boldsymbol{e}$ but opposite sign ($\boldsymbol{e}^{-}$ and $\boldsymbol{e}^{+}$, respectively)

## Electric current



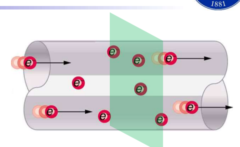
## Electric current

It is caused by moving electrons (e.g., in a circuit), ions (e.g., in a battery), or both (e.g., in plasma)



## Electric current

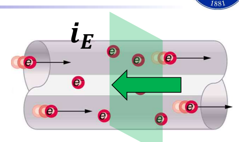
It is caused by moving electrons (e.g., in a circuit), ions (e.g., in a battery), or both (e.g., in plasma)



A flow of positive charges gives the same electric current and has the same effect in the circuit as an equal flow of negative charges in the opposite direction

## Electric current

It is caused by moving electrons (e.g., in a circuit), ions (e.g., in a battery), or both (e.g., in plasma)

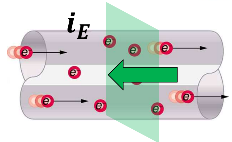



A flow of positive charges gives the same electric current and has the same effect in the circuit as an equal flow of negative charges in the opposite direction

A convention is that a **positive current** flows in the same direction as **positive charges** and vice versa

## Electric current

It is caused by moving electrons (e.g., in a circuit), ions (e.g., in a battery), or both (e.g., in plasma)

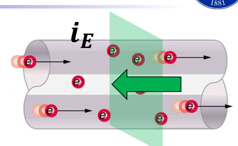



In the SI, it is measured in **Ampere** (**A**)

$1$ **A**= $1$ **C** / $1$ **s**

## Electric current

It is caused by moving electrons (e.g., in a circuit), ions (e.g., in a battery), or both (e.g., in plasma)




In the SI, it is measured in **Ampere** (**A**)

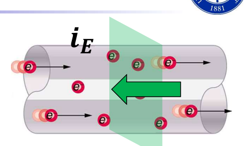
$1$ **A**= $1$ **C** / $1$ **s**

If we denote with $\Delta Q(t)$ the amount of charge that flows in the interval $[t, t + \Delta t]$, then

$$\boldsymbol{i_E}(t) = \lim_{\Delta t \to 0} \frac{\Delta Q(t)}{\Delta t} = \dot{Q}(t)$$

## Electric current

It is caused by moving electrons (e.g., in a circuit), ions (e.g., in a battery), or both (e.g., in plasma)




In the SI, it is measured in **Ampere** (**A**)

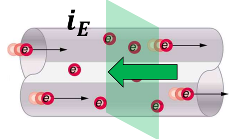
$1$ **A**= $1$ **C** / $1$ **s**

If we denote with $\Delta Q(t)$ the amount of charge that flows in the interval $[t, t + \Delta t]$, then

$$\boldsymbol{i_E}(t) = \lim_{\Delta t \to 0} \frac{\Delta Q(t)}{\Delta t} = \dot{Q}(t)$$ **It's a derivative!**

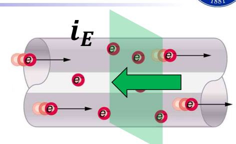
## Electric current

We usually consider two classes of electric currents:

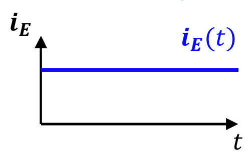



## Electric current

We usually consider two classes of electric currents:

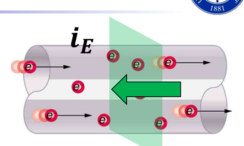



- ❑ **Direct (DC)**, i.e., the flow of charge is **unidirectional**

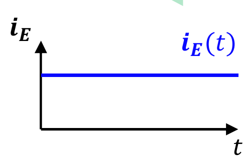



## Electric current

We usually consider two classes of electric currents:




- ❑ **Direct (DC)**, i.e., the flow of charge is **unidirectional**

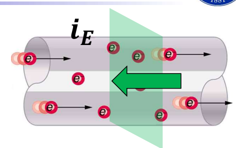


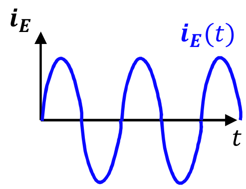

*It is produced by sources like batteries, thermocouples, solar cells, and commutator-type electric machines of the dynamo type*

## Electric current

We usually consider two classes of electric currents:

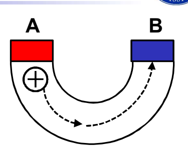
- ❑ **Direct (DC)**, i.e., the flow of charge is **unidirectional**
- ❑ **Alternating (AC)**, i.e., the movement of the charges **periodically reverses** direction



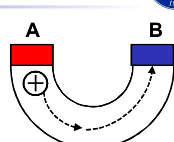

## Voltage

It is the work associated with moving a unit charge between two points along a circuit




## Voltage

It is the work associated with moving a unit charge between two points along a circuit

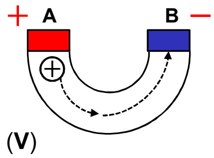



In the SI, it is measured in **Volts** (**V**)

1 **V** = 1 **J** / 1 **C**  **J** = joule

## Voltage

It is the work associated with moving a unit charge between two points along a circuit



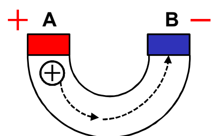

In the SI, it is measured in **Volts** (**V**)

$1\ \mathbf{V} = 1\ \mathbf{J} / 1\ \mathbf{C}$ **J** = joule

By definition, **positive charges** are pulled towards **lower voltages**. The opposite is for negative charges


## Voltage

It is the work associated with moving a unit charge between two points along a circuit



In the SI, it is measured in **Volts** (**V**)

$1\ \mathbf{V} = 1\ \mathbf{J} / 1\ \mathbf{C}$ **J** = joule

By definition, **positive charges** are pulled towards **lower voltages**. The opposite is for negative charges

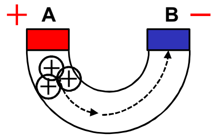

Current flows from lower voltage to higher voltage only when a source of energy “pushes” it (**battery**)

## Electric power

It is the **rate** at which work is done along a circuit

## Electric power

It is the **rate** at which work is done along a circuit




$\Delta q \stackrel{\text{def}}{=}$ amount of charge moved from A to B

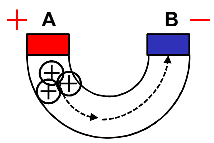
$\Delta W \stackrel{\text{def}}{=}$ work done moving $\Delta q$ from A to B

$\Delta t \stackrel{\text{def}}{=}$ time to do work $\Delta W$

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{W}}{\Delta \boldsymbol{t}}$$

## Electric power

It is the **rate** at which work is done along a circuit




$\Delta q \stackrel{\text{def}}{=}$ amount of charge moved from A to B

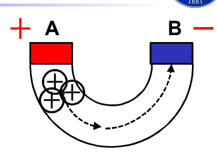
$\Delta W \stackrel{\text{def}}{=}$ work done moving $\Delta q$ from A to B

$\Delta t \stackrel{\text{def}}{=}$ time to do work $\Delta W$

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{W} \cdot \Delta \boldsymbol{q}}{\Delta \boldsymbol{q} \cdot \Delta \boldsymbol{t}}$$

## Electric power

It is the **rate** at which work is done along a circuit




$\Delta q \stackrel{\text{def}}{=}$ amount of charge moved from A to B

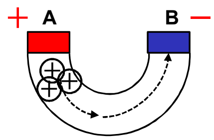
$\Delta W \stackrel{\text{def}}{=}$ work done moving $\Delta q$ from A to B

$\Delta t \stackrel{\text{def}}{=}$ time to do work $\Delta W$

$$\boldsymbol{P} = \underbrace{\boxed{\frac{\Delta \boldsymbol{W}}{\Delta \boldsymbol{q}}}}_{\textit{voltage}} \cdot \frac{\Delta \boldsymbol{q}}{\Delta \boldsymbol{t}}$$

## Electric power

It is the **rate** at which work is done along a circuit




$\Delta q \stackrel{\text{def}}{=}$ amount of charge moved from A to B

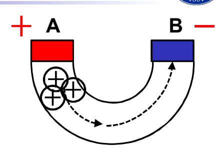
$\Delta W \stackrel{\text{def}}{=}$ work done moving $\Delta q$ from A to B

$\Delta t \stackrel{\text{def}}{=}$ time to do work $\Delta W$

$$P = \frac{\Delta W}{\Delta q} \cdot \underbrace{\boxed{\frac{\Delta q}{\Delta t}}}_{\textit{current}}$$

## Electric power

It is the **rate** at which work is done along a circuit




$\Delta q \stackrel{\text{def}}{=}$ amount of charge moved from A to B

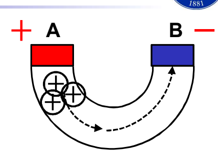
$\Delta W \stackrel{\text{def}}{=}$ work done moving $\Delta q$ from A to B

$\Delta t \stackrel{\text{def}}{=}$ time to do work $\Delta W$

$$\boldsymbol{P = v \cdot i}$$

## Electric power

It is the **rate** at which work is done along a circuit




$\Delta q \stackrel{\text{def}}{=}$ amount of charge moved from A to B

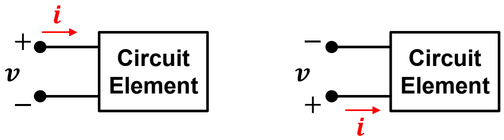
$\Delta W \stackrel{\text{def}}{=}$ work done moving $\Delta q$ from A to B

$\Delta t \stackrel{\text{def}}{=}$ time to do work $\Delta W$

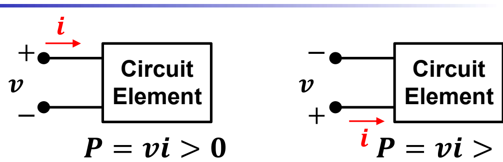
In the SI, it is measured in **Watts** (**W**)

1 **W** = 1 **J** / 1 **s** = 1 **V** × 1 **A**

## Polarity of power



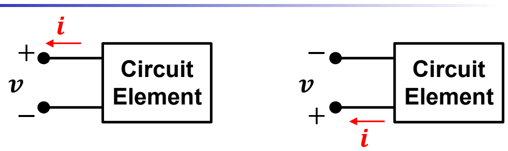

## Polarity of power



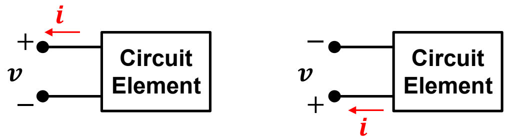

$P = vi > 0$ $\quad$ $P = vi > 0$

A positive value for power indicates that power is being absorbed (or consumed) by the circuit element

## Polarity of power



## Polarity of power




$P = -vi < 0$ $\quad$ $P = -vi < 0$

A negative value for power says that power is being generated by (or extracted from) the circuit element (e.g., **battery**)

## A circuit element is…

- **Active**

$P < 0$, i.e., it can generate energy


voltage generator


batteries

## A circuit element is…

- **Active**

$P < 0$, i.e., it can generate energy

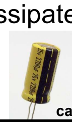

voltage generator

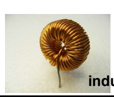

batteries

- **Passive**

$P \geq 0$, i.e., it dissipates or stores energy


resistor

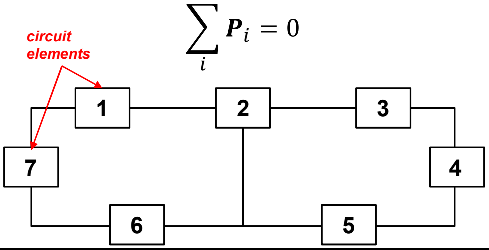

capacitor


inductor

## Remember…

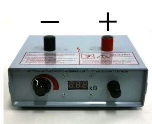
In any **closed electric circuit** the amount of power that is supplied must be equal to the amount that is absorbed

$$\sum_i \boldsymbol{P}_i = 0$$




## Source

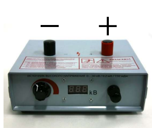
- It is an **active** circuit element with **two terminals** that provides energy to the circuit

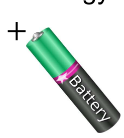




## Source

- It is an **active** circuit element with **two terminals** that provides energy to the circuit

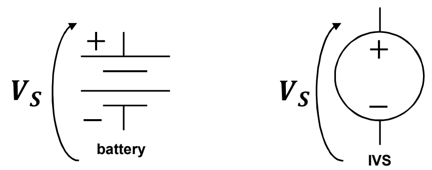



- There is no direct voltage-current ($\boldsymbol{v}$-$\boldsymbol{i}$) relationship for a source. $\boldsymbol{v}$ and $\boldsymbol{i}$ depend on the type of circuit

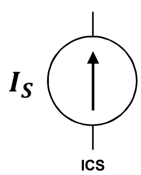
## Ideal sources

An **ideal voltage source (IVS)** generates a prescribed voltage at its terminals **regardless of the current flow**



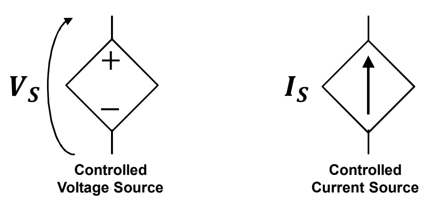

## Ideal sources

An **ideal current source (ICS)** provides a prescribed amount of current **regardless of the voltage** at its terminals

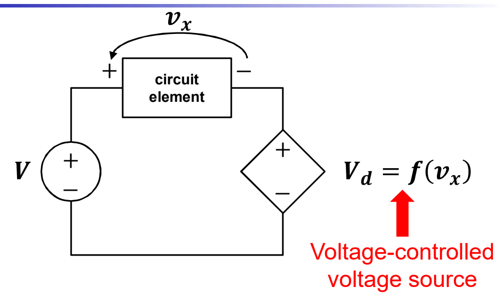



## Controlled sources

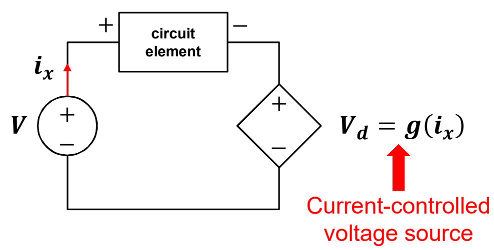
A voltage or current source is **controlled** if its output (voltage or current) depends on the voltage or current somewhere else in the circuit



## Examples




## Examples

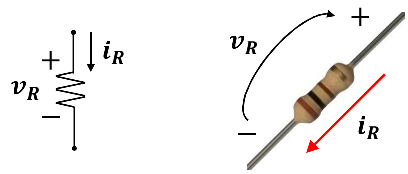



## Resistor

A circuit element that limits the flow of current through it, i.e., it opposes the current by producing a voltage drop between the terminals

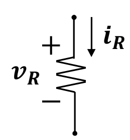
## Resistor

A circuit element that limits the flow of current through it, i.e., it opposes the current by producing a voltage drop between the terminals




## Resistor

A circuit element that limits the flow of current through it, i.e., it opposes the current by producing a voltage drop between the terminals

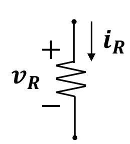



The ability to impede current is called **resistance ($R$)** and is measured in **ohms (Ω)**

1 **Ω** = 1 **V** / 1 **A**

## Resistor

A circuit element that limits the flow of current through it, i.e., it opposes the current by producing a voltage drop between the terminals

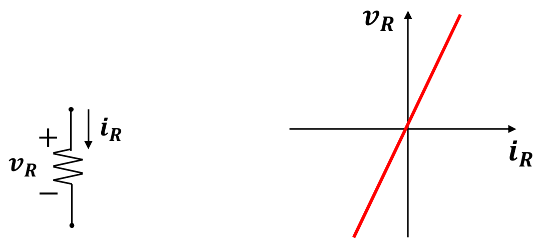



The inverse of the resistance is the **conductance ($G$)** and is measured in **siemens (S)**

1 **S** = 1 **A** / 1 **V**

## Ohm's law

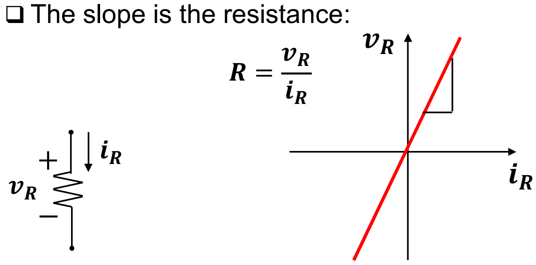
- An ideal resistor is characterized by a **linear** voltage-current relationship




## Ohm's law

- An ideal resistor is characterized by a **linear** voltage-current relationship
- The slope is the resistance:

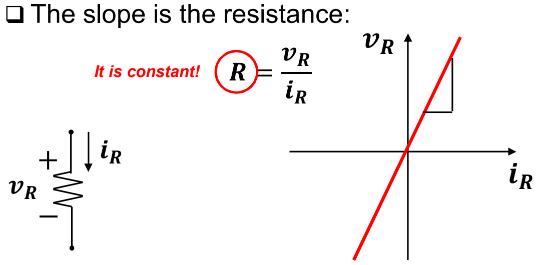
$$R = \frac{v_R}{i_R}$$




## Ohm's law

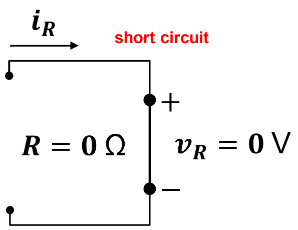
- An ideal resistor is characterized by a **linear** voltage-current relationship
- The slope is the resistance:

*It is constant!* $$R = \frac{v_R}{i_R}$$



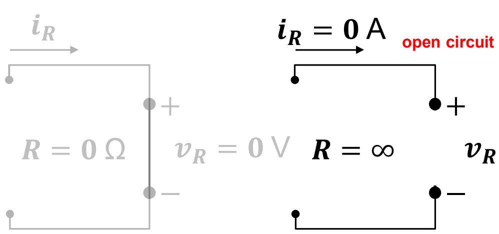
## Ohm's law ($v_R = R \cdot i_R$)

- An ideal bare wire connecting circuit elements together has resistance $R = 0\ \Omega$

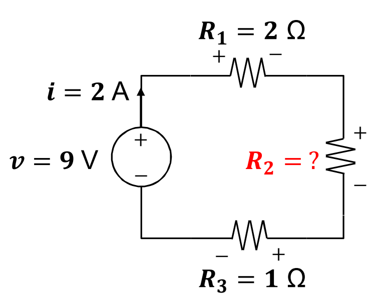



## Ohm's law ($i_R = v_R / R$)

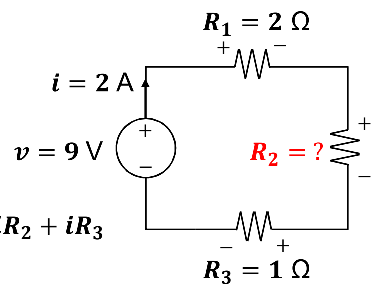
- An ideal bare wire connecting circuit elements together has resistance $R = 0\ \Omega$
- A gap between circuit elements has $R = \infty$




## An example

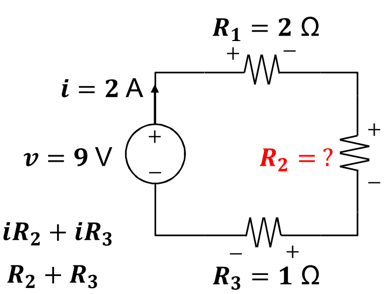


## An example



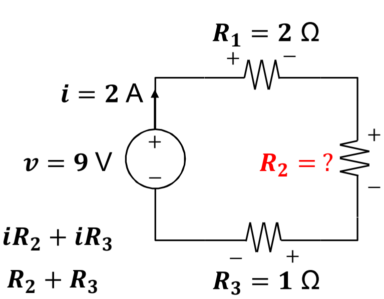

$$v = iR_1 + iR_2 + iR_3$$

## An example




$$v = iR_1 + iR_2 + iR_3$$
$$\frac{v}{i} = R_1 + R_2 + R_3$$

## An example




$$v = iR_1 + iR_2 + iR_3$$
$$\frac{v}{i} = R_1 + R_2 + R_3$$
$$R_2 = \frac{v}{i} - R_1 - R_3 = 1.5\ \Omega$$

## Power and Ohm's law

For an ideal resistor the power is:

$$P = v_R \cdot i_R = (i_R R) \cdot i_R = i_R^2 R$$

## Power and Ohm's law

For an ideal resistor the power is:

$$P = v_R \cdot i_R = (i_R R) \cdot i_R = i_R^2 R$$

- $P > 0$ independently of the signs of $i_R$ and $v_R$ (i.e., power is always absorbed)

## Power and Ohm's law

For an ideal resistor the power is:

$$P = v_R \cdot i_R = (i_R R) \cdot i_R = i_R^2 R$$

- $P > 0$ independently of the signs of $i_R$ and $v_R$ (i.e., power is always absorbed)
- The resistor dissipates the absorbed power as heat

## Exceptions to Ohm's law

- Ohm's law does not apply at very high voltages and currents (nonlinear $\boldsymbol{i}$-$\boldsymbol{v}$ relationships occur)
- Many physiological systems only follow Ohm's law in a narrow range of voltages. Outside this range the model is nonlinear
- Different materials exhibit different resistances. Some materials exhibit linear behavior over a limited range of voltage and current values