Introduction to Bioelectricity
Part I
ENGR 1166 Biomedical Engineering

What is “bioelectricity”?
- It studies the electric phenomena produced by or occurring within living organisms
- It combines Engineering and Life Sciences
- Electrical Engineering
  - Circuit theory
  - Fields theory
  - Signal processing
  - Electronics
- Life Sciences
  - Biochemistry
  - Anatomy (nerves & muscles)
  - Physiology
To measure the electric signals produced by the activity of living tissues

- Electroencephalography (EEG)
- Electroretinography (ERG)
- Electrocardiography (ECG)
- Electromyography (EMG)
Bioelectricity: areas of interest

- To measure the electric signals produced by the activity of living tissues
- To study the effect of electric fields due to an external device on tissue

- Deep brain stimulation (DBS)
- Functional stimulation (FES)
- Cardiac defibrillation (CDF)

Bioelectricity: key concepts

Electricity

The set of physical phenomena associated with the presence and flow of electric charge
**Bioelectricity: key concepts**

**Electricity**

The set of physical phenomena associated with the presence and flow of electric charge

- **Charge** \(\equiv\) A property of subatomic particles determining their electromagnetic interactions

Atoms have particles with positive (protons) and negative (electrons) charge

- **Electric current** \(\equiv\) A movement or flow of electrically charged particles through a medium
Bioelectricity: key concepts

Electricity
The set of physical phenomena associated with the presence and flow of electric charge

- **Charge** 
  A property of subatomic particles determining their electromagnetic interactions

- **Electric current** 
  A movement or flow of electrically charged particles through a medium

By definition, the direction of the current \( i_E \) is given by the direction of positive charges.

- **Voltage** 
  The work required to move a unit charge between two points
Electric charge

It is quantized, i.e., it comes in integer multiples of an individual small unit called "elementary charge" e

In the SI, it is measured in Coulomb (C)
Electric charge

It is **quantized**, i.e., it comes in integer multiples of an individual small unit called "**elementary charge**" $e$

In the SI, it is measured in **Coulomb** (C)

\[ e \equiv 1.602 \times 10^{-19} \text{ C} \]

\[ 1 \text{ C} = \frac{e}{(1.602 \times 10^{-19})} \approx 6.241 \times 10^{18}e \]

An electron and a proton have both the charge of $1e$ but opposite sign ($e^-$ and $e^+$, respectively)

Electric current
Electric current

It is caused by moving electrons (e.g., in a circuit), ions (e.g., in a battery), or both (e.g., in plasma).

A flow of positive charges gives the same electric current and has the same effect in the circuit as an equal flow of negative charges in the opposite direction.

A convention is that a positive current flows in the same direction as positive charges and vice versa.
Electric current

It is caused by moving electrons (e.g., in a circuit), ions (e.g., in a battery), or both (e.g., in plasma).

In the SI, it is measured in **Ampere (A)**

1 A = 1 C / 1 s

If we denote with $\Delta Q(t)$ the amount of charge that flows in the interval $[t, t + \Delta t]$, then

$$i_E(t) = \lim_{\Delta t \to 0} \frac{\Delta Q(t)}{\Delta t} = \dot{Q}(t)$$

It's a derivative!
We usually consider two classes of electric currents:

- **Direct (DC)**, i.e., the flow of charge is *unidirectional*.

   It is produced by sources like batteries, thermocouples, solar cells, and commutator-type electric machines of the dynamo type.
We usually consider two classes of electric currents:

- **Direct (DC)**, i.e., the flow of charge is **unidirectional**
- **Alternating (AC)**, i.e., the movement of the charges **periodically reverses** direction

**Voltage**

It is the work associated with moving a unit charge between two points along a circuit

In the SI, it is measured in **Volts (V)**

\[ 1 \text{ V} = 1 \text{ J} / 1 \text{ C} \quad J = \text{joule} \]
Voltage

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Current flows from lower voltage to higher voltage only when a source of energy "pushes" it (battery).

Electric power

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\[ \Delta q \equiv \text{amount of charge moved from A to B} \]

\[ \Delta W \equiv \text{work done moving } \Delta q \text{ from A to B} \]

\[ \Delta t \equiv \text{time to do work } \Delta W \]

\[ P = \frac{\Delta W}{\Delta t} \]
Electric power

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\( \Delta t \equiv \) time to do work \( \Delta W \)

\[
P = \frac{\Delta W}{\Delta q \Delta t}
\]

**current**

In the SI, it is measured in **Watts (W)**

1 W = 1 J / 1 s = 1 V × 1 A
A positive value for power indicates that power is being absorbed (or consumed) by the circuit element.
Polarity of power

\[ P = -vi < 0 \]

A negative value for power says that power is being generated by (or extracted from) the circuit element (e.g., battery)

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A circuit element is...

- **Active**
  
  \[ P < 0, \text{ i.e., it can generate energy} \]

- **Passive**
  
  \[ P \geq 0, \text{ i.e., it dissipates or stores energy} \]
Remember…

In any closed electric circuit the amount of power that is supplied must be equal to the amount that is absorbed

\[ \sum_{i} p_i = 0 \]

Source

- It is an active circuit element with two terminals that provides energy to the circuit
- There is no direct voltage-current (v-i) relationship for a source. \( v \) and \( i \) depend on the type of circuit
Ideal sources

An **ideal voltage source (IVS)** generates a prescribed voltage at its terminals **regardless of the current flow**

\[ V_S \]

Ideal sources

An **ideal current source (ICS)** provides a prescribed amount of current **regardless of the voltage** at its terminals

\[ I_S \]

Controlled sources

A voltage or current source is **controlled** if its output (voltage or current) depends on the voltage or current somewhere else in the circuit

\[ V_S \]
\[ I_S \]
**Examples**

\[
V_x \\
\downarrow \\
\text{circuit element} \\
\uparrow \\
V_a = f(v_x)
\]

Voltage-controlled voltage source

**Examples**

\[
V_x \\
\downarrow \\
\text{circuit element} \\
\uparrow \\
V_a = g(i_x)
\]

Current-controlled voltage source

**Resistor**

A circuit element that limits the flow of current through it, i.e., it opposes the current by producing a voltage drop between the terminals.
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\[ v_R = \frac{v}{i_R} \]

The ability to impede current is called **resistance** \((R)\) and is measured in **ohms** \((Ω)\):

\[ 1 \, Ω = 1 \, V / 1 \, A \]

The inverse of the resistance is the **conductance** \((G)\) and is measured in **siemens** \((S)\):

\[ 1 \, S = 1 \, A / 1 \, V \]
An ideal resistor is characterized by a linear voltage-current relationship.

The slope is the resistance:

\[ R = \frac{v_R}{i_R} \]

It is constant!
Ohm’s law ($v_R = R \cdot i_R$)

- An ideal bare wire connecting circuit elements together has resistance $R = 0 \, \Omega$

  $i_R$  
  \[ R = 0 \, \Omega \quad v_R = 0 \, V \]

- A gap between circuit elements has $R = \infty$

  $i_R = 0 \, A$  \[ R = \infty \quad v_R \]

An example

$i = 2 \, A$  
$s = 9 \, V$  
$R_1 = 2 \, \Omega$  
$R_2 = \ ?$  
$R_3 = 1 \, \Omega$
An example

\[ i = 2 \text{ A} \]

\[ v = 9 \text{ V} \]

\[ v = iR_1 + iR_2 + iR_3 \]

\[ R_3 = 1 \Omega \]

\[ R_2 = \text{?} \]
Power and Ohm’s law

For an ideal resistor the power is:
\[ P = v_R \cdot i_R = (i_R R) \cdot i_R = i_R^2 R \]

\[ P > 0 \] independently of the signs of \( i_R \) and \( v_R \) (i.e., power is always absorbed)

The resistor dissipates the absorbed power as heat.
**Exceptions to Ohm’s law**

- Ohm’s law does not apply at very high voltages and currents (nonlinear $i-v$ relationships occur)
- Many physiological systems only follow Ohm’s law in a narrow range of voltages. Outside this range the model is nonlinear
- Different materials exhibit different resistances. Some materials exhibit linear behavior over a limited range of voltage and current values