Recap

- **KCL**: At any node in an electrical circuit, the algebraic sum of the currents is equal to zero.
- **KVL**: The directed sum of voltages along any closed path in an electrical circuit is zero.
- **Ohm’s law**: The ratio between the voltage drop at the terminals of an ideal resistor and the current passing through it is constant and is called “resistance.”

Circuit equivalency

Two circuits are equivalent if they cannot be distinguished from each other by voltage and current measurements, i.e., if they behave identically.
In this case, since $V_S$ is applied to both circuits:

Circuit A and B are equivalent $\iff I_A = I_B$.

Sometimes it is useful to replace a complex circuit with an equivalent one that is much simpler to study.

Here we will assume that the circuits to be replaced ("reduced") are made of resistors only (i.e., passive circuits).
An example of circuit equivalency

Circuit A is any combination of resistors.

The voltage at the terminals of circuit A is $V_S$.

The circuit in red is equivalent to circuit A if

$$R_{EQ} = \frac{V_S}{I_A}$$

Resistors in series

Two resistors are in series if the same current flows from one to another.
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Two resistors in series always share one common node.

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Resistors in series

- If the same current flows through \( N \) distinct resistors then the \( N \) resistors are in series

\[ \begin{align*}
I & \quad R_1 \\
V_S & \quad + \\
 & \quad - \\
 & \quad \text{resistors in series}
\end{align*} \]

- \( R_1, R_2, \) and \( R_3 \) are in series

KVL at path 1:

\[ R_1 I + R_2 I + R_3 I - V_S = 0 \]

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V_S & \quad + \\
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 & \quad \text{path 1}
\end{align*} \]

KVL at path 1:

\[ R_1 I + R_2 I + R_3 I - V_S = 0 \]

- \( R_{EQ} = \frac{V_S}{I} = R_1 + R_2 + R_3 \)
Resistors in series

\( N \) resistors in series are equivalent to a single resistor whose resistance \( R_{EQ} \) is:

\[
R_{EQ} = \sum_{k=1}^{N} R_k
\]

KVL at path 1: \( R_1I + R_2I + R_3 I - V_S = 0 \)

Resistors in parallel

- Two or more resistors are in parallel if the same voltage is across each of them

\[
\frac{I_1}{R_1} = \frac{I_2}{R_2} = \frac{I_3}{R_3} = \frac{I}{R_{total}}
\]

Resistors in parallel

- Two or more resistors are in parallel if the same voltage is across each of them

\[
R_{1||R_2||R_3} = \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3}
\]

To denote resistors in parallel:

\[
R_1I_1 = R_2I_2 = R_3I_3 = V_S
\]
Resistors in parallel

- Resistors in parallel always share two common nodes.

\[ I \]

\[ V_s \]

\[ -I + \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} = 0 \]

KCL at node A: \(-I + I_1 + I_2 + I_3 = 0\)
Resistors in parallel

- Resistors in parallel always share two common nodes.

\[ R_{EQ} = \frac{V_s}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \]

KCL at node A: \(-I + \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} = 0\)

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Resistors in parallel

- \(N\) resistors in parallel are equivalent to a single resistor whose resistance \(R_{EQ}\) is:

\[ R_{EQ} = \frac{1}{\sum_{k=1}^{N} \frac{1}{R_k}} \]

KCL at node A: \(-I + \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} = 0\)

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Example 1

A \[ R_1 = 2 \, \Omega; \ R_2 = 6 \, \Omega; \ R_3 = 3 \, \Omega; \ R_4 = 1 \, \Omega; \ R_5 = 4 \, \Omega; \ R_6 = 12 \, \Omega \]

B

What is the equivalent circuit resistance between terminals A and B?
Example 1

\[ R_2 \parallel R_3 = 2 \, \Omega \]
\[ R_5 \parallel R_6 = 3 \, \Omega \]

Example 1

\[ R_2 \parallel R_3 = 2 \, \Omega \]
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Example 1

\[ R_4 + (R_5 \parallel R_6) = 4 \, \Omega \]
Example 1

\[ R_4 + (R_5||R_6) = 4 \, \Omega \]

Example 1

\[ \tilde{R} = (R_2||R_3)||\left( R_4 + (R_5||R_6) \right) = 4/3 \, \Omega \]

Example 1

\[ \tilde{R} = (R_2||R_3)||\left( R_4 + (R_5||R_6) \right) = 4/3 \, \Omega \]
**Example 1**

![Diagram of an electrical circuit with labels: A, B, R1, R2, R3, R4, R5, R6, and R̅.](image1)

\[ R_1 = 2 \, \Omega; \, R_2 = 6 \, \Omega; \, R_3 = 3 \, \Omega; \, R_4 = 4 \, \Omega; \, R_5 = 12 \, \Omega; \, R_6 = 1 \, \Omega; \, R_7 = 4 \, \Omega; \, R_8 = 3 \, \Omega; \, R_9 = 4 \, \Omega; \, R_{10} = 4 \, \Omega; \]

\[ R_{AB} = R_1 + \bar{R} \cong 3.33 \, \Omega \]

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**Example 2**

![Diagram of an electrical circuit with labels: A, B, R1, R2, R3, R4, R5, R6, and R7.](image2)

\[ R_1 = 2 \, \Omega; \, R_2 = 6 \, \Omega; \, R_3 = 9 \, \Omega; \, R_4 = 4 \, \Omega; \, R_5 = 4 \, \Omega; \, R_6 = 6 \, \Omega; \, R_7 = 2 \, \Omega \]

What is the equivalent circuit resistance between terminals A and B?

---

**Example 2**

![Diagram of an electrical circuit with labels: A, B, R1, R2, R3, R4, R5, R6, and R7.](image3)

\[ R_1 \parallel R_7 = 1 \, \Omega \]

\[ R_4 \parallel R_5 = 2 \, \Omega \]
Example 2

$R_1 || R_7 = 1 \, \Omega$

$R_4 || R_5 = 2 \, \Omega$

Example 2

$R_2 || R_6 = 3 \, \Omega$
Example 2

\[ R_4 = 2 \, \Omega; \quad R_2 = 6 \, \Omega; \quad R_3 = 9 \, \Omega; \quad R_4 = 4 \, \Omega; \quad R_5 = 4 \, \Omega; \]
\[ R_6 = 6 \, \Omega; \quad R_7 = 2 \, \Omega; \quad R_4||R_7 = 1 \, \Omega; \quad R_5 = 2 \, \Omega; \]
\[ R_2||R_8 = 3 \, \Omega \]
\[ \tilde{R} = R_4||R_7 + R_2||R_6 + R_4||R_5 = 6 \, \Omega \]

Example 2

\[ R_4 = 2 \, \Omega; \quad R_2 = 6 \, \Omega; \quad R_3 = 9 \, \Omega; \quad R_4 = 4 \, \Omega; \quad R_5 = 4 \, \Omega; \]
\[ R_6 = 6 \, \Omega; \quad R_7 = 2 \, \Omega; \quad R_4||R_7 = 1 \, \Omega; \quad R_5 = 2 \, \Omega; \]
\[ R_2||R_8 = 3 \, \Omega \]
\[ \tilde{R} = R_4||R_7 + R_2||R_6 + R_4||R_5 = 6 \, \Omega \]

Example 2

\[ R_4 = 2 \, \Omega; \quad R_2 = 6 \, \Omega; \quad R_3 = 9 \, \Omega; \quad R_4 = 4 \, \Omega; \quad R_5 = 4 \, \Omega; \]
\[ R_6 = 6 \, \Omega; \quad R_7 = 2 \, \Omega; \quad R_4||R_7 = 1 \, \Omega; \quad R_5 = 2 \, \Omega; \]
\[ R_2||R_8 = 3 \, \Omega; \quad \tilde{R} = 6 \, \Omega \]
\[ R_{AB} = \tilde{R}||R_3 = 3.6 \, \Omega \]
Voltage divider rule

- It allows to calculate the voltage across any individual resistor connected in a series of resistors

\[ V = \frac{V_S}{R_1 + R_2} \]

Series: \( R_{EQ} = R_1 + R_2 \)
Voltage divider rule

\[ I = \frac{V_S}{R_{EQ}} = \frac{V_S}{R_1 + R_2} \]

\[ V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V_s \]

\[ V_2 = R_2 I = \frac{R_2}{R_1 + R_2} V_s \]

- The voltage drop across each resistor in a series is a fraction of the original voltage across the entire series.

Voltage divider rule (VDR)

- If \( N \) resistors are in series \((R_1, R_2, \ldots, R_N)\) and the voltage across the series is \( V_S \), the voltage drop across each resistor is:

\[ V_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} V_S \]

\[ k = 1, 2, 3, \ldots, N \]
Current divider rule

- It allows to calculate the current through any resistor connected in parallel resistor circuits.

\[
I = I_1 + I_2
\]

Parallel: \( R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \)

Current divider rule

- It allows to calculate the current through any resistor connected in parallel resistor circuits.

\[
V = R_{EQ}I = \frac{R_1R_2}{R_1 + R_2}I
\]
The current divider rule states that in a parallel circuit, the current through each resistor is a fraction of the original current and depends on the resistance. The current through each resistor can be calculated as follows:

\[ I_1 = \frac{V}{R_1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} I \]

\[ I_2 = \frac{V}{R_2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} I \]
Current divider rule (CDR)

- If \( N \) resistors are in parallel (\( R_1, \ldots, R_N \)) and the current entering (leaving) the common nodes is \( I \), then the current through each resistor is:

\[
I_k = \frac{1}{\frac{1}{R_k} + \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} I
\]

\( k = 1, 2, 3, \ldots, N \)

Example 3

- \( R_1 = 10/3 \, \Omega; R_2 = 2 \, \Omega; R_3 = 1 \, \Omega; R_4 = 12 \, \Omega; R_5 = 12 \, \Omega; R_6 = 12 \, \Omega; I = 5 \, A \)

What is the value of \( I_2, I_5 \), and \( V_3 \)?

\[
\bar{R}_0 = R_4 || R_5 || R_6 = 4 \, \Omega
\]
Example 3

\[ R_1 = 10/3 \, \Omega; \, R_2 = 2 \, \Omega; \, R_3 = 1 \, \Omega; \, R_4 = 12 \, \Omega; \, R_5 = 12 \, \Omega; \, R_6 = 12 \, \Omega; \, I = 5 \, \text{A}; \, R_a = 4 \, \Omega \]

\[ CDR: \, I_5 = \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6} \]

\[ I_3 = \frac{1}{3} I_3 \]

Example 3

\[ R_1 = 10/3 \, \Omega; \, R_2 = 2 \, \Omega; \, R_3 = 1 \, \Omega; \, R_4 = 12 \, \Omega; \, R_5 = 12 \, \Omega; \, R_6 = 12 \, \Omega; \, I = 5 \, \text{A}; \, R_a = 4 \, \Omega \]

\[ R_b = R_3 + R_a = 5 \, \Omega \]
Example 3

**VDR:** \( V_3 = \frac{R_3}{R_3 + R_a} V \)

**CDR:** \( I_2 = \frac{R_2}{R_1 + R_2 + R_b} I \) = 0.5 \( I \) = 2.5 A
Example 3

\[ I_3 = \frac{1}{R_b + \frac{1}{R_1 + R_2}} I = 0.2 I = 1.0 \, \text{A} \]

Example 3

\[ I_5 = \frac{1}{3} I_3 \approx 0.3 \, \text{A} \]

Example 3

\[ V_3 = \frac{R_3}{R_3 + R_a} V = \frac{R_3}{R_3 + R_a} R_2 I_2 = 1.0 \, \text{V} \]