UCONN
UNIVERSITY OF CONNECTICUT

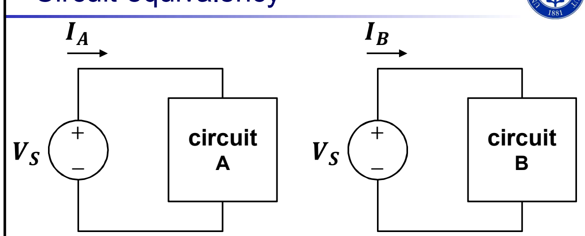
Circuit Analysis
Part I

ENGR 1166 Biomedical Engineering

Recap

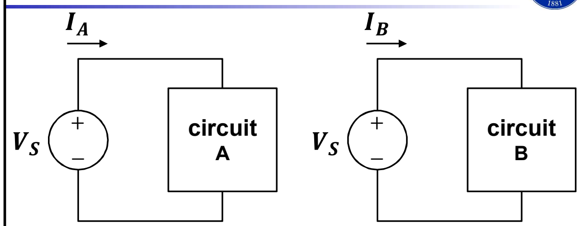
- **KCL:** At any node in an electrical circuit, the algebraic sum of the currents is equal to zero
- **KVL:** the directed sum of voltages along any closed path in an electrical circuit is zero
- **Ohm's law:** the ratio between the voltage drop at the terminals of an ideal resistor and the current passing through it is constant and is called "resistance"

Circuit equivalency



Two circuits are **equivalent** if they cannot be distinguished from each other by voltage and current measurements, i.e., if they **behave identically**

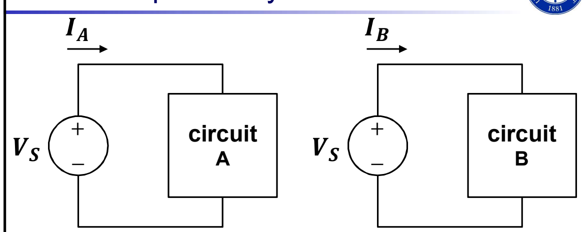
Circuit equivalency



In this case, since V_S is applied to both circuits:

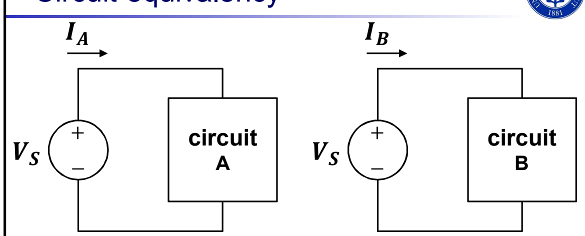
Circuit A and B are equivalent $\Leftrightarrow$ $I_A = I_B$

Circuit equivalency



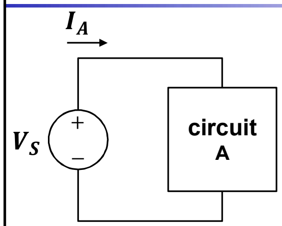
Sometimes it is useful to replace a complex circuit with an equivalent one that is much simpler to study

Circuit equivalency



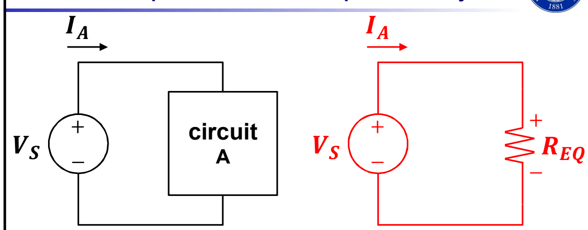
Here we will assume that the circuits to be replaced ("reduced") are **made of resistors only** (i.e., passive circuits)

An example of circuit equivalency



- Circuit A is any combination of resistors
- The voltage at the terminals of circuit A is V_S

An example of circuit equivalency

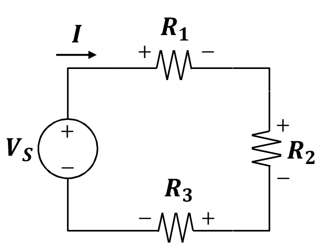


The circuit in red is equivalent to circuit A if

$$R_{EQ} = \frac{V_S}{I_A}$$

Resistors in series

- Two resistors are **in series** if the same current flows from one to another



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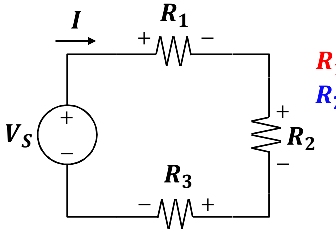
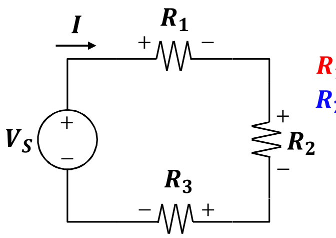


R_1 and R_2 are in series

R_2 and R_3 are in series

Resistors in series

- Two resistors in series always share **one common node**



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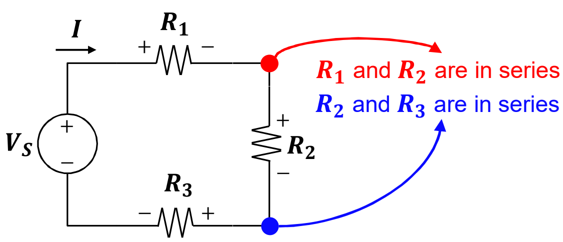
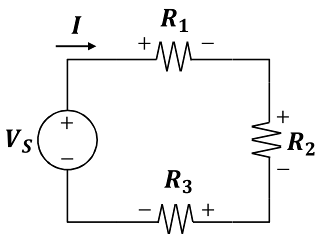


Resistors in series

- If the same current flows through N distinct resistors then the N resistors are **in series**



R_1, R_2, and R_3 are in series

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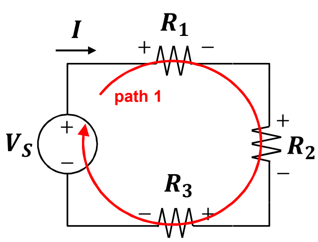

KVL at path 1: $R_1 I + R_2 I + R_3 I - V_S = 0$

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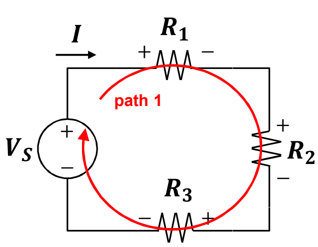


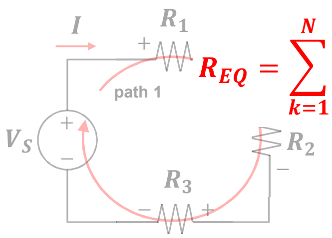
$$R_{EQ} = \frac{V_S}{I}$$
$$= R_1 + R_2 + R_3$$

KVL at path 1: $R_1 I + R_2 I + R_3 I - V_S = 0$

Resistors in series

N resistors **in series** are equivalent to a single resistor whose resistance R_{EQ} is:

$$R_{EQ} = \sum_{k=1}^{N} R_k$$



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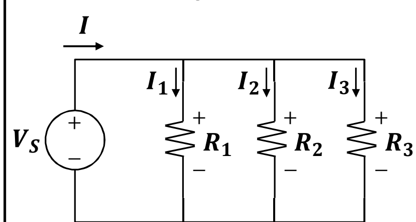
Resistors in parallel

- Two or more resistors are **in parallel** if the same voltage is across each of them



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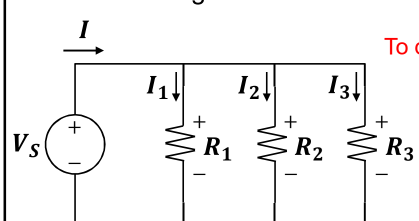


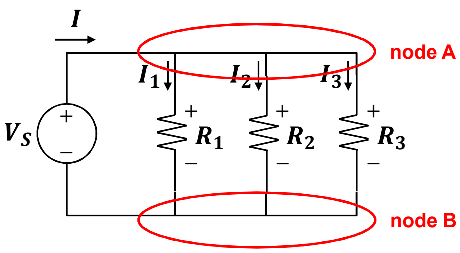
To denote resistors in parallel:

$R_1 || R_2 || R_3$

$$R_1 I_1 = R_2 I_2 = R_3 I_3 = V_S$$

Resistors in parallel

- Resistors in parallel always share **two common nodes**



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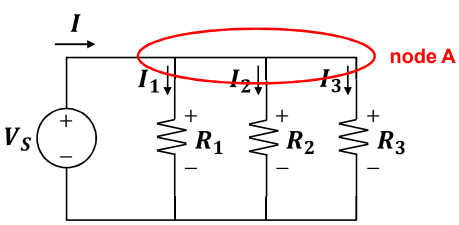

KCL at node A: $-I + I_1 + I_2 + I_3 = 0$

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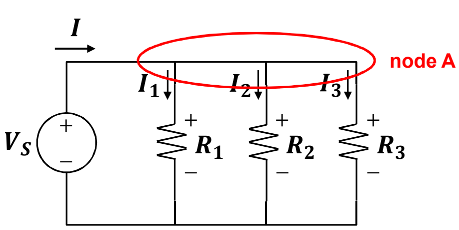

KCL at node A: $-I + \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3} = 0$

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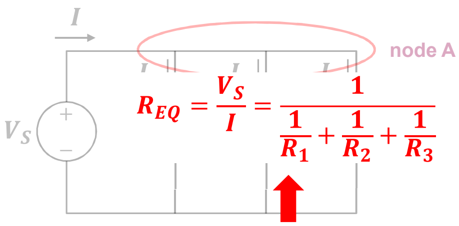

$$R_{EQ} = \frac{V_S}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

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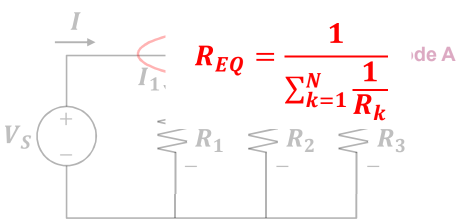

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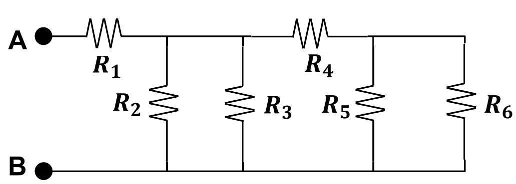
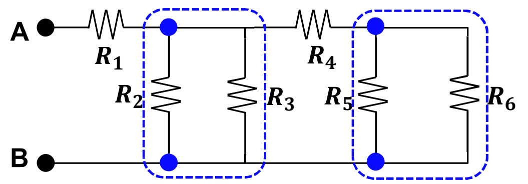


$R_1 = 2\ \Omega$; $R_2 = 6\ \Omega$; $R_3 = 3\ \Omega$; $R_4 = 1\ \Omega$; $R_5 = 4\ \Omega$; $R_6 = 12\ \Omega$

What is the equivalent circuit resistance between terminals A and B?

Example 1



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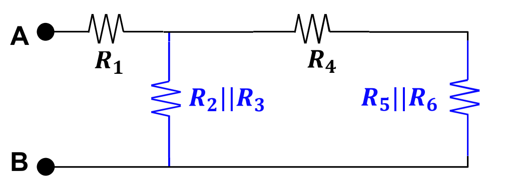

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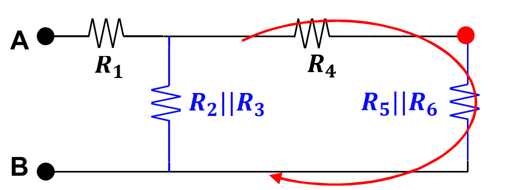

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$$R_4 + (R_5||R_6) = 4\ \Omega$$

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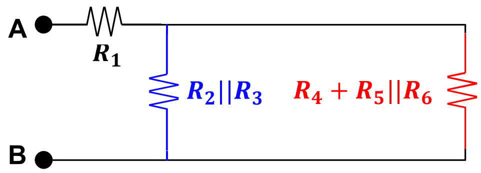

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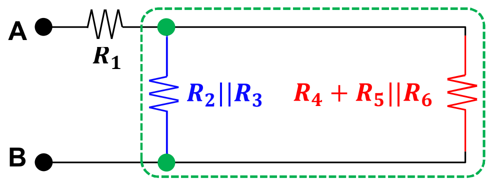

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$R_2||R_3 = 2\ \Omega$; $R_5||R_6 = 3\ \Omega$; $R_4 + R_5||R_6 = 4\ \Omega$;

$$\tilde{R} = (R_2||R_3)||\left(R_4 + (R_5||R_6)\right) = 4/3\ \Omega$$

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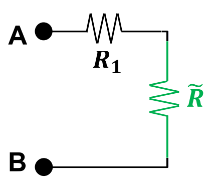

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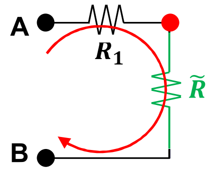

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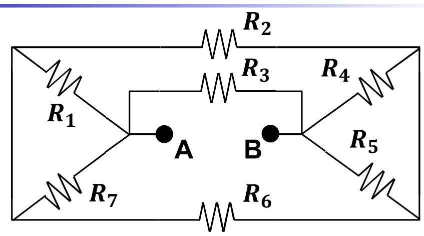
$R_2||R_3 = 2\ \Omega$; $R_5||R_6 = 3\ \Omega$; $R_4 + R_5||R_6 = 4\ \Omega$; $\tilde{R} = 4/3\ \Omega$;

$$R_{AB} = R_1 + \tilde{R} \cong 3.33\ \Omega$$

Example 2



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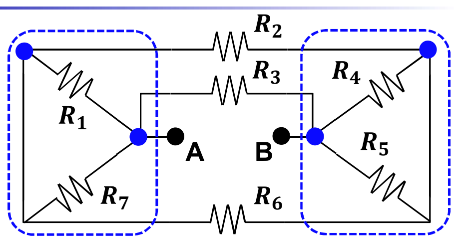
Example 2



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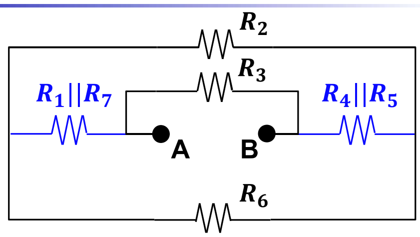

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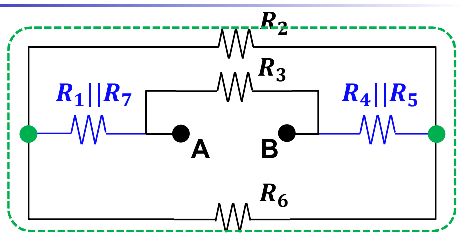

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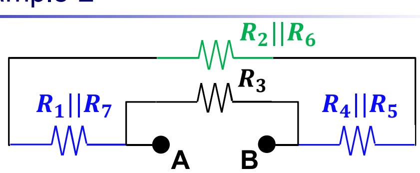

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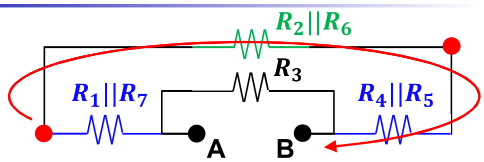

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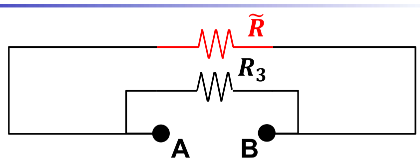
$$\widetilde{R} = R_1||R_7 + R_2||R_6 + R_4||R_5 = 6\ \Omega$$

Example 2



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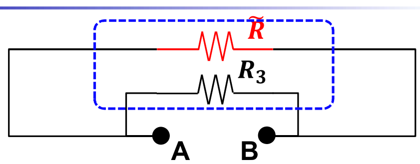

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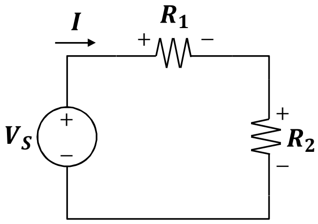
$$R_{AB} = \widetilde{R}||R_3 = 3.6\ \Omega$$

Voltage divider rule

- It allows to calculate the voltage across any individual resistor connected in a series of resistors



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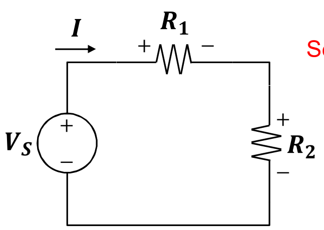

Series: $R_{EQ} = R_1 + R_2$

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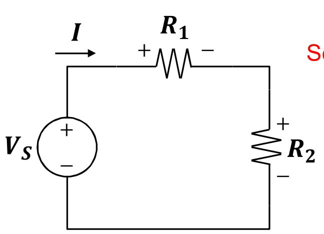


Series: $R_{EQ} = R_1 + R_2$

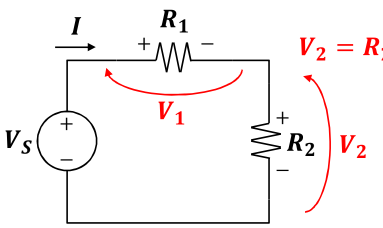
$$I = \frac{V_S}{R_{EQ}} = \frac{V_S}{R_1 + R_2}$$

Voltage divider rule

$$I = \frac{V_S}{R_{EQ}} = \frac{V_S}{R_1 + R_2}$$

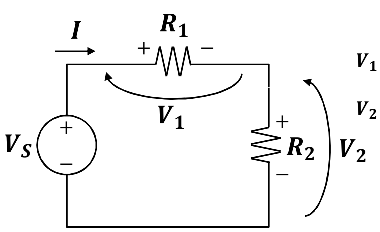
$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V_S$$

$$V_2 = R_2 I = \frac{R_2}{R_1 + R_2} V_S$$



Voltage divider rule

- The voltage drop across each resistor in a series is a fraction of the original voltage across the entire series



$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V_S$$

$$V_2 = R_2 I = \frac{R_2}{R_1 + R_2} V_S$$

Voltage divider rule (VDR)

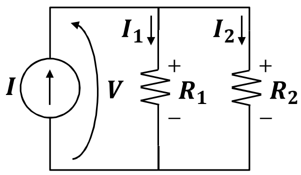
- If N resistors are in series ($R_1, R_2, \ldots, R_N$) and the voltage across the series is V_S, the voltage drop across each resistor is:

$$V_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} V_S$$

$k = 1, 2, 3, \ldots, N$

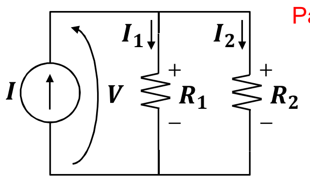
Current divider rule

- It allows to calculate the current through any resistor connected in parallel resistor circuits



Current divider rule

- It allows to calculate the current through any resistor connected in parallel resistor circuits



Parallel: $R_{EQ} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$

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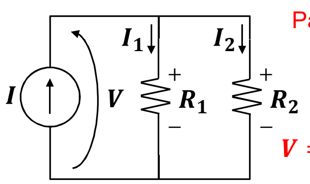

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⬇

$$V = R_{EQ}I = \frac{R_1 R_2}{R_1 + R_2} I$$

Current divider rule

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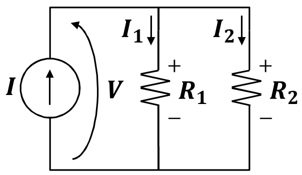
$$I_1 = \frac{V}{R_1} = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2} I$$

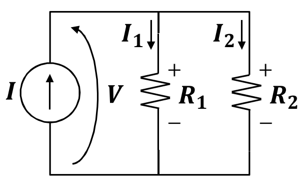


Current divider rule

$$V = R_{EQ}I = \frac{R_1 R_2}{R_1 + R_2} I$$

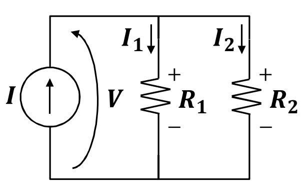
$$I_1 = \frac{V}{R_1} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} I$$

$$I_2 = \frac{V}{R_2} = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} I$$



Current divider rule

- The current through each resistor in a parallel circuit is a fraction of the original current and depends on the resistance



$$I_1 = \frac{V}{R_1} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} I$$

$$I_2 = \frac{V}{R_2} = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} I$$

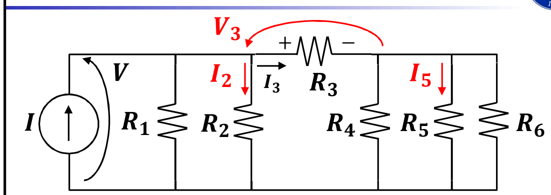
Current divider rule (CDR)

- If N resistors are in parallel ($\boldsymbol{R_1},\ldots,\boldsymbol{R_N}$) and the current entering (leaving) the common nodes is $\boldsymbol{I}$, then the current through each resistor is:

$$\boldsymbol{I_k} = \frac{\frac{1}{R_k}}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{R_N}}\boldsymbol{I}$$

$\boldsymbol{k = 1, 2, 3, \ldots, N}$

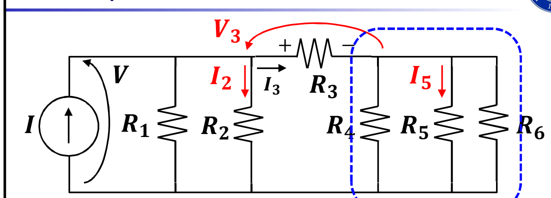
Example 3



$\boldsymbol{R_1} = \mathbf{10/3}\ \Omega$; $\boldsymbol{R_2} = \mathbf{2}\ \Omega$; $\boldsymbol{R_3} = \mathbf{1}\ \Omega$; $\boldsymbol{R_4} = \mathbf{12}\ \Omega$; $\boldsymbol{R_5} = \mathbf{12}\ \Omega$; $\boldsymbol{R_6} = \mathbf{12}\ \Omega$; $\boldsymbol{I} = \mathbf{5}$ A

What is the value of I_2, I_5, and V_3?

Example 3



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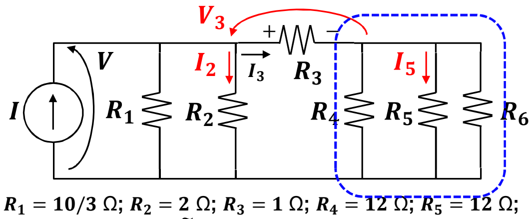
$$\widetilde{\boldsymbol{R}}_{\boldsymbol{a}} = \boldsymbol{R_4} || \boldsymbol{R_5} || \boldsymbol{R_6} = \mathbf{4}\ \Omega$$

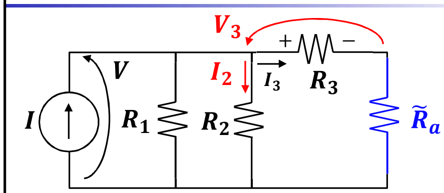
Example 3



$R_1 = 10/3\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $R_4 = 12\ \Omega$; $R_5 = 12\ \Omega$; $R_6 = 12\ \Omega$; $I = 5$ A; $\tilde{R}_a = 4\ \Omega$

CDR: $I_5 = \dfrac{\frac{1}{R_5}}{\frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_6}} I_3 = \frac{1}{3} I_3$

Example 3



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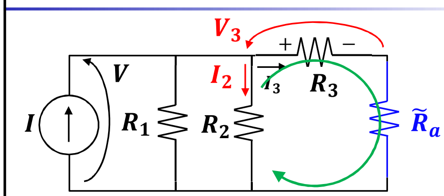
Example 3



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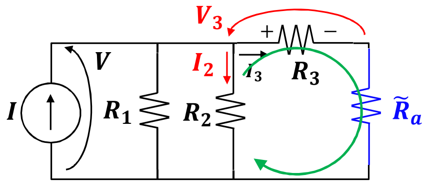
$$\tilde{R}_b = R_3 + \tilde{R}_a = 5\ \Omega$$

Example 3



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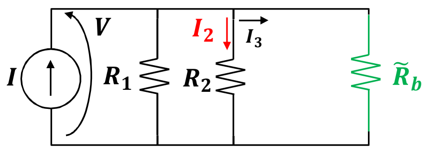
$R_6 = 12\ \Omega$; $I = 5$ A; $\tilde{R}_a = 4\ \Omega$; $\tilde{R}_b = 5\ \Omega$

VDR: $V_3 = \frac{R_3}{R_3 + \tilde{R}_a} V$

Example 3



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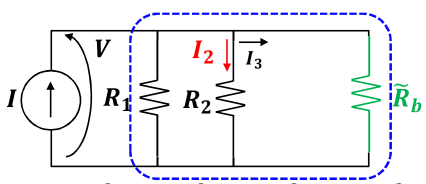
Example 3



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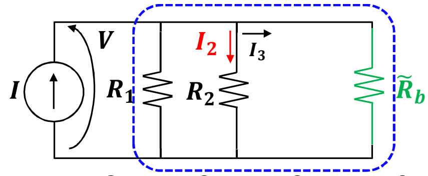
$R_6 = 12\ \Omega$; $I = 5$ A; $\tilde{R}_a = 4\ \Omega$; $\tilde{R}_b = 5\ \Omega$

CDR: $I_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{\tilde{R}_b}} I = 0.5I = 2.5$ A

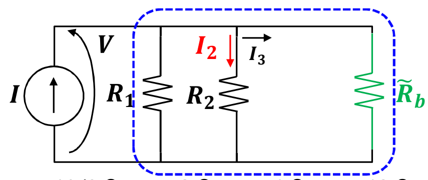
Example 3



$R_1 = 10/3\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $R_4 = 12\ \Omega$; $R_5 = 12\ \Omega$;
$R_6 = 12\ \Omega$; $I = 5$ A; $\tilde{R}_a = 4\ \Omega$; $\tilde{R}_b = 5\ \Omega$

CDR: $I_3 = \frac{\frac{1}{\tilde{R}_b}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{\tilde{R}_b}} I = 0.2I = 1.0$ A

Example 3



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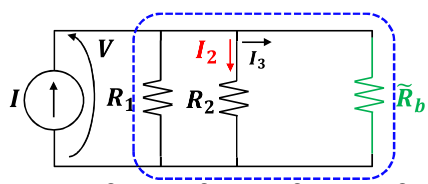
$R_6 = 12\ \Omega$; $I = 5$ A; $\tilde{R}_a = 4\ \Omega$; $\tilde{R}_b = 5\ \Omega$

CDR: $I_5 = \frac{1}{3} I_3 \cong 0.3$ A

Example 3



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$R_6 = 12\ \Omega$; $I = 5$ A; $\tilde{R}_a = 4\ \Omega$; $\tilde{R}_b = 5\ \Omega$

VDR: $V_3 = \frac{R_3}{R_3+\tilde{R}_a} V = \frac{R_3}{R_3+\tilde{R}_a} R_2 I_2 = 1.0$ V